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ANALYZING THE EFFECT OF WATER SEEPAGE AND SCOUR ON SLOPE STABILITY USING FINITE ELEMENT METHOD

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ABSTRACT

On every slope, there is always the possibility of landslides. One of the hypotheses tested was the effect of water flow and scour on slope stability at the investigated location. A brief inspection was carried out to determine the condition and influence of the river, in addition to soil investigations, for proper follow-up arrangement. At landslide sites, water flow has an effect in raising the groundwater level so as to reduce the soil shear strength. Its causes the slope safety factor to drop below the safe rate. The scouring disrupts the stability of the slope but does not reduce the slope safety factor significantly. The parametric studies are chosen for their strong relation between measured displacement within the slope, to verify the measurements and the numerical analysis (Finite Element Method) results. The parametric study simulation shows that time-dependent numerical model could be useful as forecasting tool, including various possible scour and non-scour condition.

Keywords: Groundwater Table, Finite element Method (FEM), Landslides, Parametric Studies, Multilaminate Model, Creep/Viscoplastic

ABSTRAK. ANALISIS PENGARUH ALIRAN AIR DALAM TANAH DAN GERUSAN TERHADAP STABILITAS LERENG MENGGUNAKAN METODE ELEMEN HINGGA. Di setiap lereng, selalu ada kemungkinan longsor. Salah satu hipotesis yang diuji adalah pengaruh aliran air dan gerusan terhadap stabilitas lereng di lokasi yang diteliti. Inspeksi singkat dilakukan untuk mengetahui kondisi dan pengaruh sungai, selain penyelidikan tanah, untuk pengaturan tindak lanjut yang tepat. Pada lokasi longsor, aliran air berpengaruh dalam menaikkan muka air tanah sehingga dapat menurunkan kuat geser tanah. Hal ini menyebabkan faktor keamanan lereng turun di bawah angka aman. Gerusan tersebut mengganggu kestabilan lereng tetapi tidak mengurangi faktor keamanan lereng secara signifikan. Studi parametrik dipilih karena hubungannya yang kuat antara perpindahan terukur di dalam lereng, untuk memverifikasi pengukuran dan hasil analisis numerik (Methode Elemen Hingga). Simulasi studi parametrik menunjukkan bahwa model numerik yang bergantung pada waktu dapat berguna sebagai alat peramalan, termasuk berbagai kemungkinan kondisi gerusan dan non gerusan.

Kata kunci*: Muka Air Tanah, Metode Elemen Hingga, Longsoran lereng, Studi parametrik, Multilaminate Model, Rayapan/Viscoplastic*

1. INTRODUCTION

Two events occurred in a nearby location. First, landslides that occur on the road to the bridge that shown in Figure 1(a), and it has disrupted the movement of vehicles crossing the bridge. Second, landslides that occur near the location of a factory, and make the factory building columns shift as in Figure 1(b). Stakeholders need inspections to find out about the situation and the effects of the river in addition to the soil investigations, so better arrangement follow-up efforts can be made.

(a) The bridge embankment landslides (b) Shifting column

Figure 1. Landslides and It's Effect Nearby the Location

This event was apparently not the first. Landslides at these two points have occurred and action has been taken to repair them. At the factory site, repairs have been carried out by strengthening the 9-meter piles and stone gabions on the riverbank. At the bridge location, the wingwall of the bridge has shifted and fallen into the river body. The embankment approach has been strengthened with a retaining wall as deep as 4 meters, but the landslide movement continues. Thus, it is necessary to review the details of the cause. One of the assumptions that need to be tested is the influence of the flow and scour of the water on the stability of the river slopes in both locations.

Slope is a surface that connects higher ground with lower soil surface, and slope stability is closely related to landslides or ground motion which is the process of transferring soil mass naturally from higher to lower places. While landslides occur because the imbalance of forces acting on the slope or the force on the slope area is greater than the restraining force in the slope. On every slope, there is always the possibility of landslides. Landslides occur due to driving forces exceeding the opposing forces produced by soil shear strength along landslide fields (Das, 1985). Technically it can be said that landslides occur if the safety factor (SF) does not meet, SF <1.5 (Table 25 SNI Geoteknik 8460:2017).

Clear-water scour is scouring that occurs if there is no movement of basic material (no material is transported) upstream of the building, or theoretically $\tau_0 < \tau_c$ where τ_0 is the shear strength that occurs, while τ_c is the critical shear strength of the riverbed grains. Scouring with sedimentary water is scouring that occurs when flow conditions in a river cause riverbed material to move or theoretically $\tau_0 > \tau_c$ where the shear strength in the channel is greater than its critical value (Chow, et al., 1988). The scour depth balance is fulfilled when the amount of moved material from the scour hole is equal to the material supplied into it. This balance is hard if the water level changes frequently, coupled with changing speeds.

Based on these things, the problem faced is how to describe the effect of water flow and scouring on landslides that occur in the two locations. For bridge case, furthermore, the slope stability analysis proceeds, by using Finite Element Method #1, Limit Equilibrium Method, Finite Element Method #2, with an implementation of the Multilaminate-Model, named *Newmo3962* (Louhenapessy 2000, Louhenapessy & Pande 2000). These methods and criteria above, are undertaken to do assessment and identify the slope stability problems. Thus, this paper is intended to provide a brief description of the possible causes of landslides. All descriptions given are preliminary conclusions because the study work is still in the pre-DED (before Detailed Engineering Drawing) stage.

2. METHODOLOGY

2.1 Introduction

The research method used is descriptive explanatory method (Kothari, 2004). This method contains several steps: (1) selecting and formulating the problem, (2) determining the objectives of the study, (3) providing scope of research, (4) exploring the sources of relevant literature, (5) formulating hypotheses to be tested, both explicitly and implicitly, (6) doing field work (in-situ), (7) providing interpretation of results in relation to the conditions to be investigated, and (8) conducting generalizations and deductions from the findings and hypotheses tested.

The hypothesis to be tested is the flow and scour of water have a significant influence on the instability of river slopes as in Figure 2.

On the software used in the analyses:

For Finite Element Model #2, the software use is Newmo3962 (Louhenapessy & Pande 2000), this software is the update of the older generation of the Multilaminate model algorithm by Zienkiewicz & Pande (1977). The Newmo3962 is equipped with the following numerical computational capabilities:

a. Time dependent analysis in this case: visco-plastic analysis of the soil (Zienkiewicz & Cormeau 1974, Swoboda et.al. 1987, Louhenapessy 2000).
b. Undrained analysis (Louhenapessy 2000).

The theory and equation behind the FEM Multilaminate model can be found in Section 2.2 and elsewhere (Zienkiewicz & Cormeau 1974, Swoboda et.al. 1987). Other paper describing the finite element method (Bargess et.al 2019) and limit equilibrium method is by Harianto Rahardjo et.al. (2016).

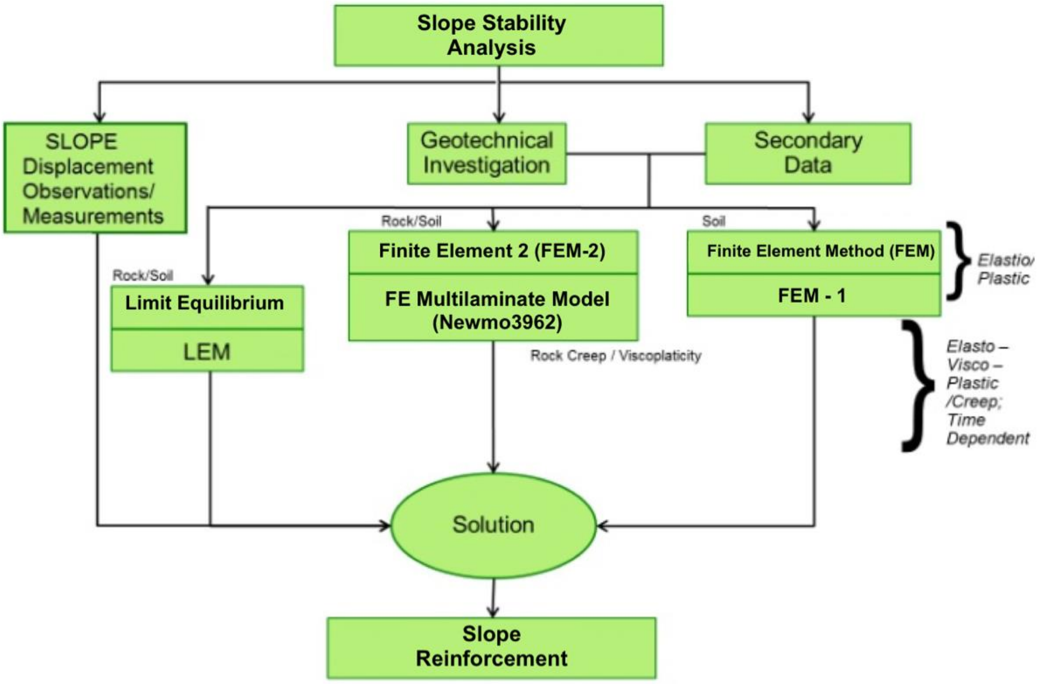


Figure 2. Flow Chart Slope Stability and Reinforcement Strategy (Louhenapessy, 2017) – Rock Creep, meaning it should add soil creep too in the flow chart

2.2 Multilaminate Concept: Viscoplasticity for Soil and Rock

Due to the absence of the characteristic rheological values of most types of soil and rock masses needed for viscoplastic calculations, the author decided to use the simplest possible viscoplastic flow rule for the purpose of back-analysing measured values. The simplest viscoplastic flow rule is the one-dimensional mechanical model shown in Figure 3. Strain is composed of an elastic and a viscoplastic component:

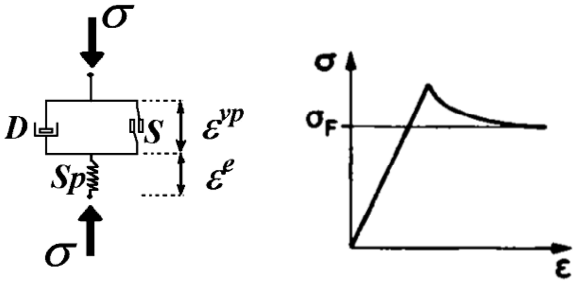
$$\{\varepsilon\} = \{\varepsilon_e\} + \{\varepsilon_{vp}\} = \{\varepsilon_e\} + \{\dot{\varepsilon_{vp}}\}.\Delta t \quad (1)$$

with

$$\{\varepsilon_e\} = [D]^{-1}.\{\sigma\} \quad (2)$$

The model's stress-strain diagram can be described as follows: if stresses occur that are smaller than the yield stress of the St. Venant element σ_F , no ε_{vp} is permitted. This means that the strains are purely elastic. If the stresses σ exceed the yield stress σ_F , the St. Venant element (Slider) can only receive σ_F and permits additional infinite strains. These are received by the Newton element (Dashpot), whereby the following holds true:

$$(\sigma - \sigma_F) = \eta.\frac{\partial\varepsilon}{\partial t} = \eta.\dot{\varepsilon} \quad (3)$$



LEGEND:
D = Dashpot
S = Slider
Sp = Spring strains
ε^e = Elastic strains
ε^{vp} = viscoplastic strains

Stresses σ imposed stresses on the rock mass

Figure 3. Rheological Mechanical Model for Soil and Rock
(Rheological analogue of elasto-viscoplastic: Bingham Model)

The time-dependent strain on the Newton element causes the strain on the Hook element to drop and thus, according to (2), the stress σ in the structure, provided that the overall system strain remains constant. At time t = ∞, σ drops to σ_F.

$$\{\dot{\varepsilon_{vp}}\} = \frac{\{\varepsilon_{vp}\}}{\partial t} = \gamma . \left\langle \Phi\left(\frac{F}{F_o}\right)\right\rangle . \frac{\partial Q}{\partial\{\sigma\}} \quad (4)$$

The second term contains the flow function *F*, as well as F_O, convenient reference value of *F*. Only by using F_O, do we finally determine $\{\dot{\varepsilon_{vp}}\}$ in its dimensionally pure state. Φ is a creep function empirically found from creep test. The use of an exponential law is shown:

$$\Phi\left(\frac{F}{F_o}\right) = \left(\frac{F}{F_o}\right)^n \quad (5)$$

The following functions are used as the yield condition F according to (4):

$$\textit{Mohr-Coulomb} \qquad F = \frac{J_1}{3}.sin\varphi + J_2^{0.5}\left(cos\beta - \frac{sin\varphi sin\beta}{\sqrt{3}}\right) - c.cos\varphi \quad (6)$$

The reference value was given the following values:

$$\textit{Mohr-Coulomb} \qquad F_o = c.cos\varphi \quad (7)$$

Whereby J_1, J_2 and J_3 are the stress invariants, c the cohesion and ϕ the angle of friction. β is Lode's angle.

3. SITE INVESTIGATION

3.1. The Bridge Case

There is usually an influence of the type of abutments and bridge pillars on the flow (Chow et al. 1988, Melville, 2015). But in this case, there was no such effect. The flow exits from the culvert and falls on the river after the bridge, at least 30 m^3 / sec (when 6 culverts are full). The riverbed is not strengthened, so this flow erodes it. The wing wall on both sides of the bridge had collapsed several years before and had not been fully cleaned.

The remnants are big enough to direct the flow as in Figure 4(a). Thus, the flow turns sideways and erodes the bottom of the bridge embankment slope that shown in Figure 4(b).

(a) The remnants of the bridge wing wall in the river.

(b) The downstream turns sideways and erodes the bottom of the bridge embankment slope

Figure 4. River Condition.

Scouring power is obtained from the narrowing of the cross-section. The riverbed had been eroded, in a narrow width. Because the discharge is the same, as consequences, the flow will be faster. The bottom slope will erode, and the embankment slope becomes unstable.

However, the soil's small collapse due to scoured river flow did not conduct landslides, because its position was slightly from the far end of the collapse. It is necessary to look for other causes, suspected to be seepage of water flow on the side of the bridge. At the side and bottom of the bridge, there are 2 kinds of water flowing from the gutter. The first stream goes straight into the u-ditch box and flows downstream. The second flow flows and seeps between the u-ditch wall and the abutment wall that shown in Figure 5.

This flow, because it is clamped by two concrete fields, has the potential to raise the groundwater level. Changes in groundwater level will change the mechanical properties of the soil, especially the shear strength. The surface of the road embankment is at + 9.80 m from the original river surface. So, when it rains normally for a long duration, this flow can affect the strength of the bottom of the embankment.

Figure 5. Flow at the Bottom of the Bridge

Observations on the right bank of the river, especially the shifting of the retain poles (4 meters depth) mounted on the riverbank, show that the landslide pressure is quite high, and the slide is deeper than the bottom of the pole. While the top end of the pole (left of the river) tilts toward the river shows that there is a scouring effect so that the poles lose the strength of the anchoring. Thus, a new retaining wall with sufficient depth is needed.

If the retaining wall is designed assuming the soil in front of the wall to be as high as a riverbed is considered to be non-existent, then the river only needs to be cleansed of material and repaired edges for straight flow downstream. If only repairing and adding the retaining poles (with insufficient depth), it is necessary to extend the U-ditch with a designed wall that functions as a retaining wall with a height of the wall that can accommodate high flood discharge flow. This option requires the calculation of flood flow discharge.

As this study were limited by time and equipment, the effect of unsaturated soil conditions was minor and not in priority. Seepage is assumed only for saturated soil conditions. In the future research, the author will take into account additional literatures for the unsaturated soils (i.e., Satyanaga et.al. 2021 and Pande & Pietruszczak 2015).

3.2. The Factory Case

In the river near the factory, the smallest river width is around 14 meters. In the dry season, it is only 4 meters wide, and the edges are overgrown with shrubs. River cross-section is relatively flat, but at certain times the flow can have a depth of 2-3 meters. So, the flow that occurs is at least 56 m^3/second (when the river overflows). The curve of the river resembles a horseshoe as in Figure 6. The type is Meander. The soil is easily eroded by the flows (Marhendi, 2018). Thus, the discharge is sufficient to erode riverbanks.

The upper slope has been reinforced with concrete piles (9-meter depth) and the bottom has been reinforced with gabions, while the riverbed is not strengthened. The stream hits the right bank of the river and is forced to turn, and erodes the river slope, and further disrupts the stability of the riverbank. While the left side will be filled with sediment ramps. The scour goes slowly (creep).

Figure 6. The factory-side river flow (left), streams erode the river slopes on the side of the factory and reduce the retaining power of gabions (middle), and the condition of concrete piles (right)

In Figure 6, the alignment of a river that curves and erodes the side of a factory. Dolken poles and gabions shift towards the middle of the river. The top of the pole tilted toward the river. Shifted due to landslide pressure. However, from the appearance, the top of the concrete pile is tilted towards the factory. So, the bottom has shifted towards the river. Instead, the dolken poles on the riverbank on the upper side tilt toward the river.

The slope of concrete piles indicates that the movement of the soil in the lower layer is huge. Thus, the depth of the piles has not yet reached the solid layer. While the slope of the poles indicates that the riverbed is eroded so that the pole loses its retaining force and is pushed by the landslide load. Thus, the pole and gabion were pushed into the river body.

The cross section of the river at low tide (low water level) will decrease and as consequences, the current will be faster, thereby increasing scouring power. Meanwhile, when the water level is high (during the rainy season or flood), the water meets the cross section of the river, reaching the front side of the concrete piles. This causes the groundwater level to rise. Soil water content increases thereby reducing shear strength (Handayani, et al., 2014). The soil under pressure will move through the pile and even make the concrete pile shift. While the movement of gabions is caused by the pressure of the landslide material and the scouring effect.

Based on this, scouring does occur and will cause landslides if the riverbank slopes are not strengthened properly. But scour is not the main cause. An increase in groundwater level due to river water level is one of the factors, in addition to the load generated by factory activities. Thus, if the riverbank alongside the factory is strengthened by a retaining wall with sufficient depth, then the stability of the slope against the scour can be restrained by stone gabion construction. It only needs to be improved and increased in length, which is the length of the landslide field. In addition (if possible), to protect the slopes against scouring, river alignment needs to be straightened.

4. RESULTS AND DISCUSSION

4.1. Soil Parameters

Geotechnical design parameters are interpreted based on the results of a combined interpretation of laboratory and field tests (in-situ). The testing procedure is based on ASTM Standards. A summary of the results can be seen in Table 1.

Table 1. The Laboratory Test Result

Desc.	Soil	Unit Weight	Compressibility Parameter			Undrained Parameter		Drained Parameter	
			C_c	C_v	C_r	c	ϕ	c'	ϕ
		γ (kN/m³)		cm²/s		kPa	º	kPa	º
Layer 1	Clay (CH)	17	0.29	7.32E-03	0.04	30	15	-	19
Layer 2	Clay (CH)	17	0.6 - 0.81	0.00659 to 0.00859	0.10-0.15	90	(5.41, use 0.0)	-	19
Layer 3	Clay CH, MH /OH	17	-	-	-	210	30	-	19

4.2. Geotechnical Modeling (Finite Element Method)

The following step will be the Finite Element Method or FEM-2 (accompanied by capability to simulate creep / time-based creep or Time-Step). The FEM-2 is used to model the cross section of the bridge as in Figure 7 and the Finite Element model as in Figure 8, where there were 147 elements and 498 nodal points.

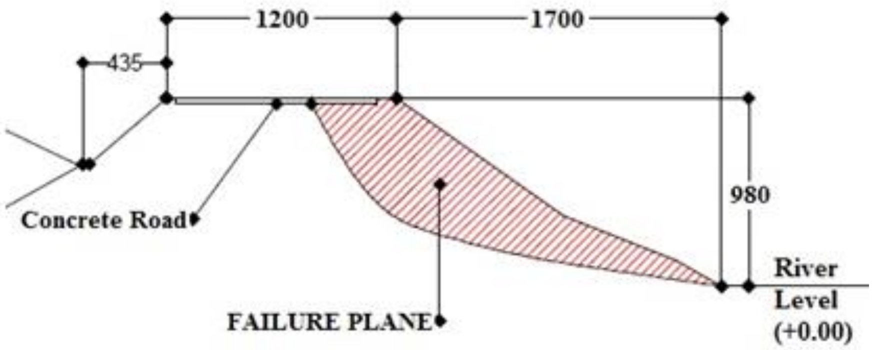


Figure 7. Cross Section C2 of the Bridge

4.3. Results

The horizontal displacement of points on the slopes (points 71, 112, 205, 472) displaced up to 70 cm; At the right-hand side of Figure 9, shows the result, in the form of deformed shaped. Depicted the horizontal and resultant displacements of the bridge's cross section slope. Where the right-hand side does not have support / pinned (modelling assuming the scour), hence produce a deformation up to 1.2 m as in Figure 10.

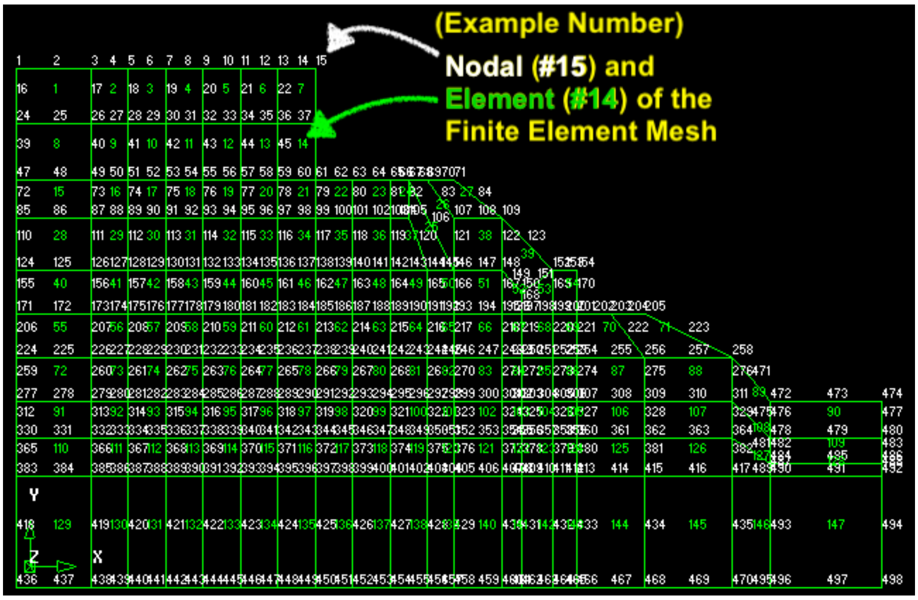


Figure 8. FEM-2 Mesh Modelling with 147 Elements and 498 Nodal Points

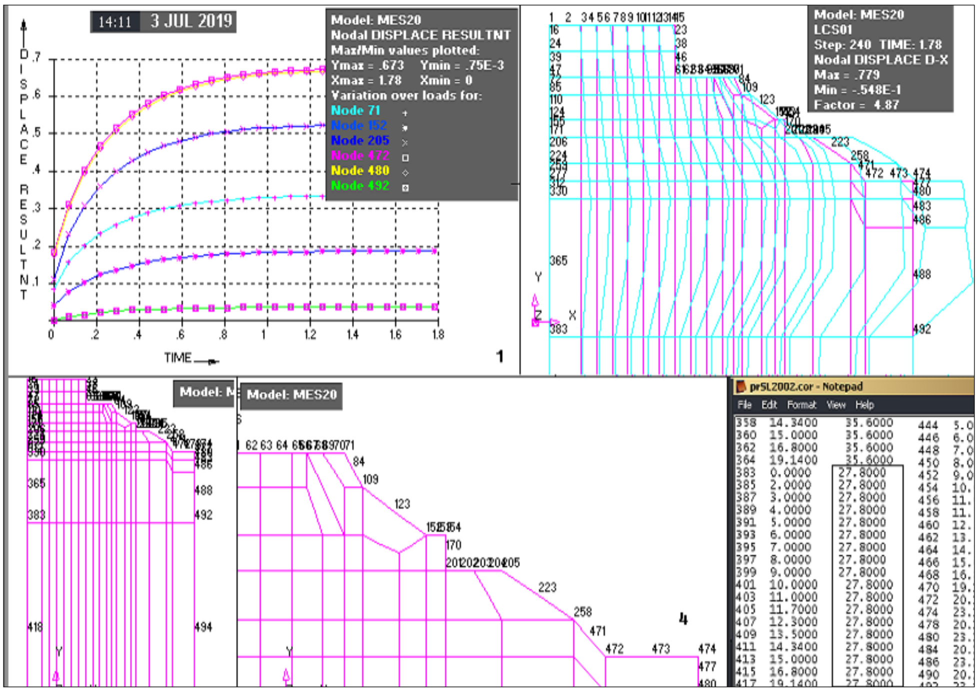


Figure 9. Horizontal Displacement of Points on the Slopes (points 71, 112, 205, 472) Displaced Up to 70 cm; Top Right: The Results, in the Form of the Deformed Shaped

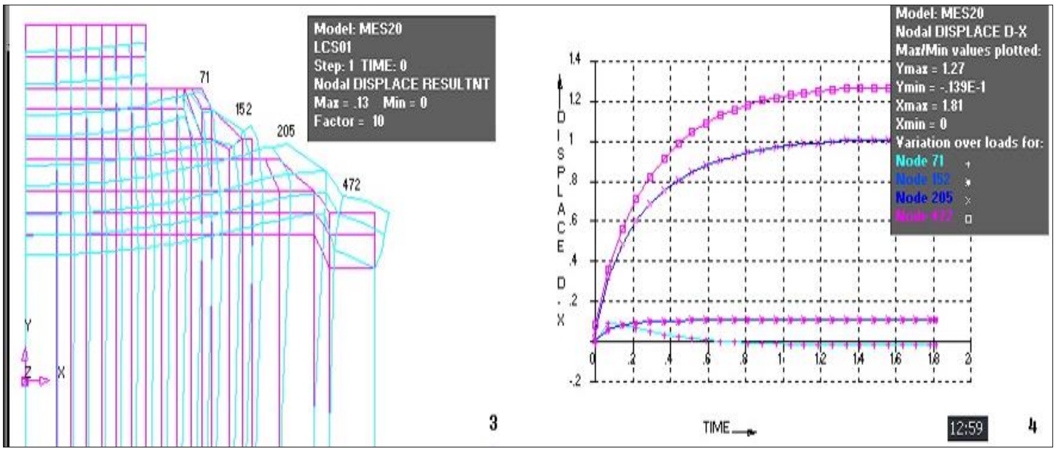


Figure 10. Horizontal and Resultant Displacement of the Bridge's Cross Section Slope. Where the right-hand side does not have support/pinned (modelling assuming the scour), hence produce a deformation up to 1.2 m

In Figure 11, depicted the vertical stress (S-YY) when elastic and the yield condition (failure). Maximum concentrated stress appears in element #53 (at the middle of the slope); and depicted the Displacement Resultant and Principal Stresses (S-11) in Finite Element modelling, with and without a Soil Retaining Wall as in Figure 12.

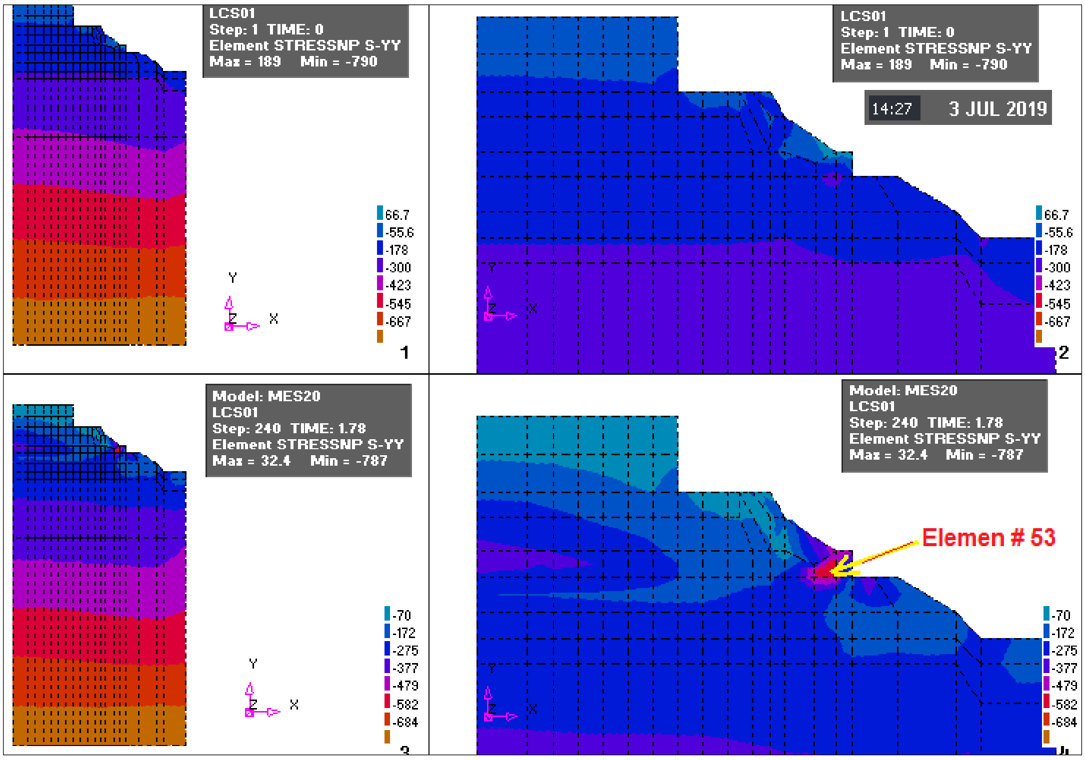
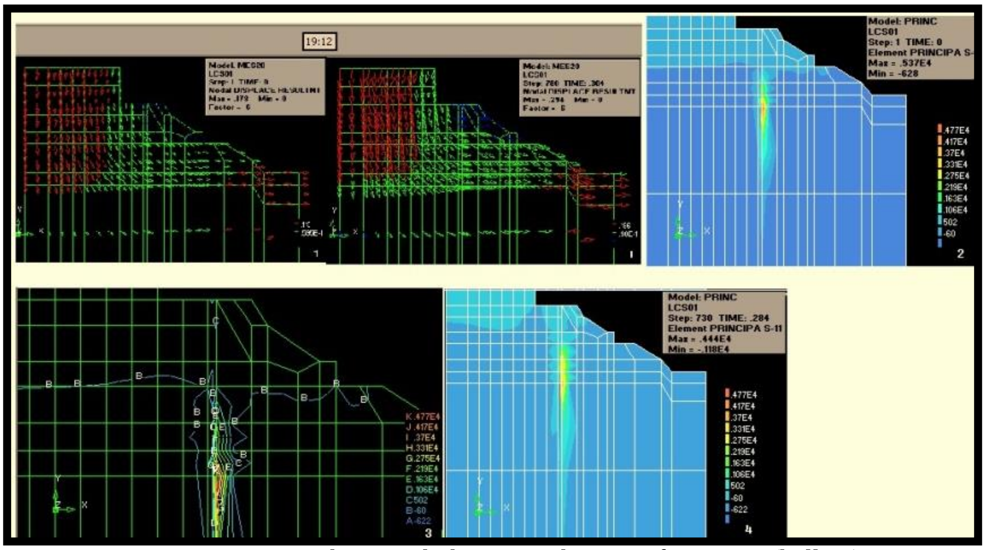


Figure 11. Vertical Stress (S-YY) in Elastic and the Yield Condition (failure). Maximum concentrated stress appears in element #53 (at the middle of the slope).



Compare above with the one without reinforcement (bellow)

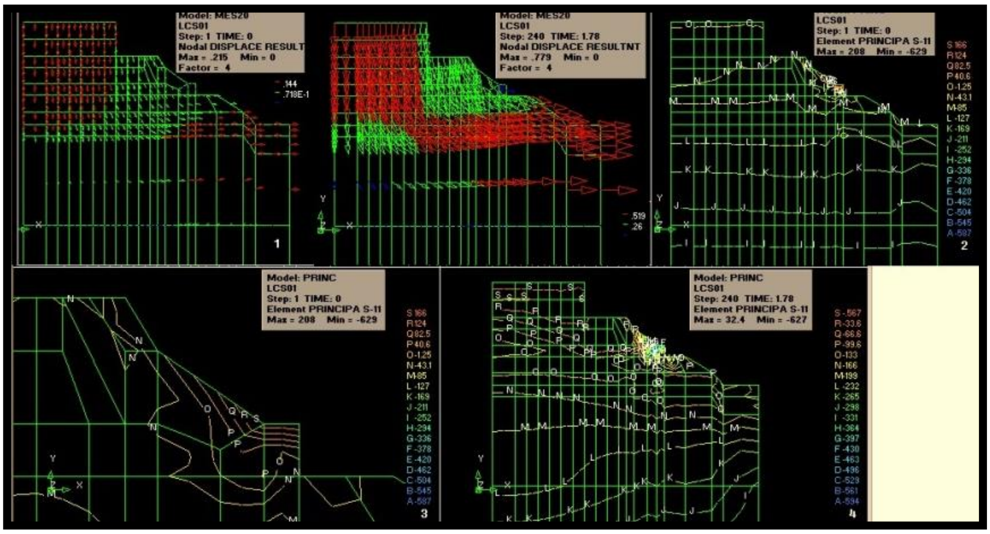


Figure 12. Displacement Resultant and Principal Stresses (S-11) in FEM, with and without a Soil Retaining Wall

5. CONCLUSIONS

The stability of the slope to scour due to the influence of water flow needs to be considered. At the bridge location, a significant influence is the flow of water that seeps into the bottom of the road embankment. Scouring only makes it worse. It is necessary to build a retaining wall with sufficient depth. The river must be cleaned for smooth flow.

Based on the model's output, the following remarks can be summarized:

a. The Parametric Studies are chosen for their strong relation between measured displacement within the slope, to verify the measurements[1] (Lareno et.al. 2019) and the numerical analysis

[1] As in Figure 5, through the site observation/measurement, lateral displacement is 75 cm.

(FEM) results. The Parametric Study simulation shows that a time-dependent numerical model could be useful as a forecasting tool.

b. In the analysis of Finite Element (both Elastic and Multilaminate Model-Newmo3962) the parametric study has been carried out including various possible scour and non-scour conditions (with removing the right-hand pin/support at the right geometry of the slope; at the toe).

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