MODE I NON-LINEAR FRACTURE MODEL:
CASES ON CONCRETE AND FIBER REINFORCED CONCRETE

Rr. M.I. Retno Susilorini
Lecturer, Department of Civil Engineering, Faculty of Engineering,
Soegijapranata Catholic University
Jl. Pawiyatan Luhur IV/1, Bendan Dhuwur, Semarang 50234
E-mail: retno_susilorini@yahoo.com; susilorini@unika.ac.id

ABSTRACT

Mode I non-linear fracture model has been used widely to derive the values of stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$ as fracture criterions for for concrete and fiber reinforced concrete. Some previous ode I non-linear fracture models of are *Fictitious Crack Model* by Hillerborg, (1976), *Crack Band Model* by Bazant (1983, 1986), "Two-Parameters Model" by Jenq and Shah (1986), "Mode I Crack Propagation Model" by Zhang and Li (2005), and "Non-local Damage Model" by Ferrara and Prisco (2005). This paper implements the theories of mode I non-linear fracture model on 2 cases. One case is implemented to concrete and another case is implemented to fiber reinforced concrete. Those two cases will derive the values of stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$ for each case. Case 1 is a case of concrete (without fiber) notched beam specimen of mode I non-linear fracture model and Case 2 is infinite fiber reinforced concrete of mode I non-linear fracture model. Case 1 resuls the values of stress intensity factor K^S_{Ic} as 15.078 MPa mm$^{-1/2}$ and crack tip opening displacement $CTOD_c$ as 0.023 mm. Case 2 resuls the values of stress intensity factor K^S_{Ic} as $3.917.10^{-4}$ MPa mm$^{-1/2}$ and crack tip opening displacement $CTOD_c$ as $-1.994.10^{-4}$ mm. In general, the fiber existence gives a great influence in deriving the analytical solution. This paper meet the conclusion as follows: (1) Mode I non-linear fracture model can be used to derive the values of stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$ as fracture criterions for for concrete and fiber reinforced concrete, (2) The fracture behavior of fiber reinforced concrete is specific compare to concrete because the existence of fiber bridging phenomenon, (3) Calculating stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$, the result will be over-estimated if fiber traction is ignored, and it could be under-estimated if FPZ is ignored, and (4) It is a good solution to combine Case 1 and Case 2 calculation together to get the values of stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$ by considering the existence of fiber in fiber-matrix composites.

Keywords: mode I, non-linear, fracture, model, concrete, fiber reinforced concrete.

ABSTRACT

Model fraktur ragam I non-linier telah banyak digunakan untuk memperoleh faktor intensitas tegangan K^S_{Ic} dan perpindahan bukaan ujung retak $CTOD_c$ sebagai kriteria fraktur untuk beton dan beton serat. Beberapa model fraktur ragam I non-linier terdahulu antara lain *Model Retak Fiktif* oleh Hillerborg, (1976), *Model Pita Retak* oleh Bazant (1983, 1986), *Model Dua-Parameter* oleh Jenq dan Shah (1986), *Model Penjalaran Retak Mode I* oleh Zhang dan Li (2005), dan Model Kerusakan Non-Lokal oleh Ferrara dan Prisco (2005). Tulisan ini mengimplementasikan model fraktur ragam I non-linier pada 2 kasus. Kasus pertama diimplementasikan pad beton sedangkan kasus kedua diimplementasikan pada beton serat. Kedua kasus tersebut akan memperoleh nilai faktor intensitas tegangan K^S_{Ic} dan perpindahan bukaan ujung retak $CTOD_c$. Kasus 1 adalah kasus benda uji balok beton bertakik model fraktur ragam I non-linier dan kasus 2 adalah beton serat tak hingga model fraktur ragam I non-linier. Kasus 1 menghasilkan nilai faktor intensitas tegangan K^S_{Ic} sebesar 15.078 MPa mm$^{-1/2}$ dan perpindahan bukaan ujung retak $CTOD_c$ sebesar 0.023 mm. Kasus 1 menghasilkan nilai faktor intensitas tegangan K^S_{Ic} sebesar $3.917.10^{-4}$ MPa mm$^{-1/2}$ dan perpindahan bukaan ujung

retak $CTOD_c$ sebesar $-1.994.10^{-4}$ mm. Secara umum, keberadaan serat sangat mempengaruhi solusi analitis. Tulisan ini memperoleh kesimpulan sebagai berikut: (1) Model fraktur ragam I non-linier dapat digunakan untuk memperoleh faktor intensitas tegangan K^S_{Ic} dan perpindahan bukaan ujung retak $CTOD_c$ sebagai kriteria fraktur untuk beton dan beton serat, (2) Perilaku fraktur beton serat adalah spesifik dibandingkan beton karena adanya fenomena penjembatanan serat, (3) Dalam perhitungan hasil faktor intensitas tegangan K^S_{Ic} dan perpindahan bukaan ujung retak $CTOD_c$ akan berlebihan bila traksi serat diabaikan dan kurang bila Zona Proses Fraktur diabaikan, (4) Akan sangat baik bila mengkombinasikan Kasus 1 dan Kasus 2 bersama-sama untuk memperoleh nilai faktor intensitas tegangan K^S_{Ic} dan perpindahan bukaan ujung retak $CTOD_c$ dengan memperhatikan keberadaan serat dalam komposit matriks berserat.

Kata kunci: ragam I, non-linier, fraktur, model, beton, beton serat.

1. INTRODUCTION

Crack on concrete is associated seldomly with the growth of micro cracks at crack-tip zone which is described by inelastic zone or *fracture process zone* (FPZ). The FPZ becomes obstacle in implementing *Linear Elastic Fracture Mechanics* (LEFM) on cementitious materials (Shah et. al, 1995). The obstacles is that LEFM doesn't take account stable crack growth related to FPZ, or in another words it can be said that there is small FPZ in LEFM, therefor the initial crack length is used to determine the critical stress intensity factor. Contrary to the fact, there is a relative large microcracking zone exists adjacent the crack front (Zhang and Li, 2005). Covering the problem, the *Non-Linear Fracture Mechanics* (NLFM) is chosen as more appropriate tool in modeling the concrete fracture.

Failure on cementitious materials has shown the formation and growth of cracks (Chern et.al, 1989). In case of fiber reinforced concrete, the fibers that are distributed on brittle materials will against the formation and growth of cracks. It is pointed by fiber bridging that increases the fiber-matrices bond mechanism and also the fiber stiffness around the crack surface.

Ratanalert dan Wecharatana (1989) noted some previous classical non-linear models which are applied to opening mode of fracture mechanics (Broek, 1982), generally stated as "mode I". The models mentioned above are (for example): *Fictitious Crack Model* by Hillerborg, 1976, *Crack Band Model* by Bazant, 1983, 1986, and "Two-Parameter Model" by Jenq and Shah, 1986. However, there are other latest models, for examples, "Mode I Crack Propagation Model" by Zhang and Li (2005) and "Non-local Damage Model" by Ferrara and Prisco (2005).

Considering some models mentioned above, this paper will review the mode I non-linear fracture model by implementing the theories on two cases of concrete and fiber reinforced concrete to derive fracture parameters of stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$.

2. LITERATURE REVIEW

2.1 Elastic Linear Fracture Mechanics

Linear Elastic Fracture Mechanics (LEFM) assumed that all fracture process happens at crack-tip while all the body volume remains elastic (Bazant, 1992). Based on this assumption, the crack growth and structural failure will be solved by elastic-linear method. According to elasticity theory, stress near by the crack-tip is close to infinity eventhough the load apllied is very small (Bazant, 1992) then the failure is considered by energy criterion, not strength criterion. While the crack-tip is growing, the energy flows to crack-tip and dissipated by fracture process. The critical crack growth is postulated by Griffith as:

$$G = G_f \tag{1}$$

where:

G = fracture energy (J/m^2 or N/m)
G_f = critical fracture energy (J/m^2 or N/m)

For $G < G_f$, crack will not grow, for $G > G_f$, the equilibrium will not be reached.

The energy released rate for mode I expressed by:

$$G_I = \frac{K_I^2}{E} \tag{2}$$

where:

G_I = energy released rate mode
K_I = stress intensity factor mode I
E = modulus of elasticity

Critical crack growth on mode I defined by stress intensity factor, that is:

$$K_I = K_{Ic} \tag{3}$$

where:

K_{Ic} = critical value on K_I,
     defined as fracture toughness which represents material properties
     for $K_{Ic} = G_f E$

It is noted that K_{Ic} is difficult to be determined, that is indirectly determined by calculating G_f.

2.2 Quasi-Brittle Non-Linear Fracture Mechanics

The stress-strain curve for ideal brittle materials will be elastic-linear until reach the maximum stress. The different curve trend will be found on quasi-brittle material such as

concrete, that is the non-linearity which is happened before the maximum stress is reached. The mechanism after proportional limit, f_y, cannot be fully understood (Shah et. al, 1995). On initial stage, micro cracks are distributed randomly. On some points before peak stress, micro cracks being localized to be macro cracks which grow critically while peak stress is reached. Strain-softening will occur on steady-state crack growth. On post-peak stage, the displacement accompanies the opening of major cracks with unloading condition.

Fracture of concrete's behaviour is dominated by FPZ. The FPZ existance generates significant energy dissipation (Balaguru and Shah, 1992). It is understandable that initial notch length cannot be accepted as critical crack measurement because the crack will propagate in stable condition until the effective crack length be reached. In cases of fiber reinforced concrete, the fibers contribute to load-carrying capacity as well as increase the composites toughness which influences the formation of FPZ.

The LEFM is not considered for direct implementation to concrete or other cementitious materials because of particles bridging and FPZ variation along the thickness (Shah and McGarry, cited by Balaguru dan Shah, 1992). Therefore, accurate description of concrete fracture should be presented such as parts of crack and also inelastic respon of material in FPZ. The application of fracture mechanics on concrete failure is currently covered by models of fracture mode I simulations on effective crack line (Shah and McGarry, cited by Balaguru and Shah, 1992). FPZ variation along the thickness is commonly ignored.

Inelastic fracture response caused by FPZ existence is considered as cohesive pressure that is applied on crack surface (Jenq and Shah, 1989). The FPZ toughening mechanism is modeled as cohesive pressure on crack surface. Cohesive pressure (w) is a monotonic decreasing function of crack separation displacement, w. The cohesive pressure (w) has same value with material tension strength f_t for w = 0 on crack tip. This phenomenon is implied that micro crack in front of the crack is not included in FPZ. It can be understood if the size of crack tip of FPZ is smaller than the crack growth zone.

Since concrete with quasi-brittle crack suffers load, the effective quasi-brittle crack tip produces energy released rate G_q. The energy released rate G_q divided into 2 parts: (1) Energy rate during the fracture process of 2 surfaces, G_{Ic}, that is ekivalent to material surface energy, and (2) Energy rate to cope the cohesive pressure (w) that separates the surfaces, G_σ, by showing the parts of energy which will open the crack. Then, the energy released rate G_q for quasi-brittle crack of mode I defined as:

$$G_q = G_{Ic} + G_\sigma \tag{4}$$

The value of G_{Ic} would be evaluated by LEFM and stated as critical energy released rate. When the value of G is the same as the cohesive pressure work for a unit of crack length for a structure with a unit thickness, then the value of G will be expressed by:

$$G_\sigma = \frac{1}{\Delta a} \int_0^{\Delta a} \int_0^w \sigma(w) dx dw = \frac{1}{\Delta a} \int_0^{\Delta a} dx \int_0^w \sigma(w) dw = \int_0^{w_t} \sigma(w) dw \qquad (5)$$

where:

$\sigma(w)$ = normal cohesive pressure

w_t = crack separation displacement at initial crak tip

If the part of dx is taken out of the integral, then the equation (5) explains that form of crack opening displacement w does not vary with crack length change. It is noted that equation (5) remains accurate when materials at crak tip remain touching each other. When crack separation is too bigger that some of crack surfaces have been separated, then $w_t > w_c$, and the upper limit on equation (5) will be replaced by w_c. Hence, the term of w_c is stated as critical crack separation displacement where crack separation reaches (w) = 0.

The w_t will become smaller than w_c on peak load. Substituting equation (5) to equation (4), the expression (6) will be defined as:

$$G_q = G_{Ic} + \int_0^{w_t} \sigma(w) dw \qquad (6)$$

According to equation (6), it should be remembered that the integral upper limit w_t must be replaced by w_c when $w_t > w_c$. Equation (6) will remains valid when there are two forces, they are applied force and cohesive force, that are applied into FPZ. Therefor, equation (6) represents general energy equilibrium for quasi-brittle crack of mode I growth. It is also emphasized that equation (6) indicates that energy released rate G_q of quasi-brittle fracture influenced by 2 mechanisms of fracture energy dissipation. Those 2 mechanisms are: (1) Griffith-Irwin energy dissipation mechanism that is represented by energy released rate G_{Ic}, and (2) Dugdale-Barenblatt energy dissipation mechanism that is represented by material tension force G . It should be noted that energy dissipation mechanisms can be used aquivalent elastic crack approach for non-linear fracture modeling.

2.3 Two-Parameters Fracture Model by Jenq and Shah

Previous section emphasizes that modeling the FPZ can be approached by Griffith-Irwin energy dissipation mechanism. The model assumes that (w) = 0, as adopted by

equivalent-elastic crack approach. According to Shah, et. al (1995), some models used LEFM criterion and equivalency between actual crack and effective crack that was explicitely determined. Hence, energy released rate for mode I effective elastic crack is defined as well as:

$$G_q = G_{Ic} \tag{7}$$

where:

G_q = function of geometry, structural size, and applied load (i.e. effective-elastic crack)

G_{Ic} = critical energy released rate (i.e. material fracture toughness)

It is shown by equation (7) that effective-elastic crack grows with the applied load change increase. When the increase of crack length on stable crack growth grows with the applied load change, there must be another equation needed before calculation of equation (7). Empirically, effective-elastic crack length is independent with geometry and structural size. It can be understood that effective-elastic crack length can't be used as independent fracture criterion. Therefor, another fracture criterion should be implemented in the calculation.

A proper procedure may be used to distinguish whether the fracture criterion component is elastic or plastic. A specimen is loading until maximum stress $_c$ reached by. The next step, unloading is implemented followed by reloading. According to compliance of unloading, value of CMOD (*Crack Mouth Opening Displacement*) at peak load $CMOD_c$, will be divided into elastic and plastic components, and expressed by:

$$CMOD_c = CMOD_c^e + CMOD_c^p \tag{8}$$

where:

$CMOD_c$ = CMOD value at peak load

$CMOD_c^e$ = elastic $CMOD_c$ component

$CMOD_c^p$ = plastic $CMOD_c$ component

Measured values from procedure mentioned above, $CMOD_c^e$ and $_c$ will be substituted into equations of LEFM to derive critical stress intensity factor K^S_{Ic} and effective-elastic crack length a_c that is defined by:

$$K_{Ic}^s = \sigma_c \sqrt{\pi a_c}\, g_1\left(\frac{a_c}{b}\right) \tag{9}$$

$$CMOD_c^e = \frac{4\sigma_c a_c}{E} g_2\left(\frac{a_c}{b}\right) \qquad (10)$$

The value of CTOD^e_c can be determined by CMOD^e_c, σ_c, and a$_c$, by using equation:

$$CTOD_c^e = CMOD_c^e\, g_3\left(\frac{a_c}{b}, \frac{a_0}{a_c}\right) \qquad (11)$$

where: K_{Ic}^S = critical stress intensity factor

     a_c = maximum stress
     a_c = effective-elastic crack length
     a_0 = initial crack length
     CMOD^e_c = elastic $CMOD_c$ component
     E   = elasticity modulus
     CTOD^e_c = critical crack tip opening displacement
     g_1, g_2, and g_3 = geometrical function for each different specimen
                            (Shah, et. al, 1995)

Jenq and Shah (1985, cited in Shah, et. al, 1995) proposed the two-parameters fracture model based on elastic fracture response of structure. Their experimental test resulted that beams with different sizes but consisted of same material had constant values of K_{Ic}^S and CTOD^e. According to their experimental result, Jenq and Shah proposed critical fracture properties of quasi-brittle material that are stated as values of K_{Ic}^S and CTOD^e. For any certain structural material with different size and geometry that is applied by critical fracture load (in this case, peak load) will meet 2 criterions as follow:

$$K_I = K_{Ic}^S \text{ and } CTOD = CTOD_C \qquad (12)$$

where: K$_I$ = stress intensity factor

     CTOD   = crack tip opening displacement

     K$^S_{Ic}$ = critical stress intensity factor

     CTOD$_c$ = critical crack tip opening displacement

2.4 RILEM Method by Jenq and Shah for K$^S_{Ic}$ and CTOD$_c$ Calculation

RILEM Technical Committee 89-F (1990) proposed a recommendation to measure fracture parameters of stress intensity factor K$^S_{Ic}$ and crack tip opening displacement CTOD$_c$ by using three-point flexural beam. The proposed method is based on two-parameter fracture model by Jenq and Shah (Shah et. al, 1995). The calculation procedure to derive K$^S_{Ic}$ and CTOD$_c$ are described as follow:

(1) Plot the load-CMOD curve of specimen

(2) Calculate *compliance* C_i and C_u. The values of C_i derived by intial stage of the load-CMOD curve, and the values of C_u derived by unloading stage of the load-CMOD curve

(3) Calculate elasticity modulus, E, at initial stage by using equation:

$$E = \frac{6 S a_c g_2(\alpha_0)}{C_i b^2 t} \tag{13}$$

where:

E	= elasticity modulus
S	= beam span
$g_2(\alpha_0)$	= geometry function
C_i	= *compliance* of intial stage of the load-CMOD curve
b	= beam's depth
t	= beam's width

(4) The values of $g_2(\alpha_0)$ can be derived by:

$$g_2(\alpha_0) = 0.76 - 2.28\alpha_0 + 3.87\alpha_0^2 - 2.04\alpha_0^3 + \frac{0.66}{(1-\alpha_0)^2} \tag{14}$$

$$\alpha_0 = \frac{(a_0 + HO)}{(b + HO)} \tag{15}$$

where:

a_0	= initial notch depth
b	= beam's depth
HO	= clipper gage thickness

(5) Calculate elasticity modulus, E, at unloading stage by using equation:

$$E = \frac{6 S a_c g_2(\alpha_c)}{C_u b^2 t} \tag{16}$$

where:

E	= elasticity modulus
S	= beam span
$g_2(\alpha_0)$	= geometry function
C_u	= *compliance* of unloading stage of the load-CMOD curve
b	= beam's depth
t	= beam's width

(6) The values of $g_2(\alpha_c)$ can be derived by:

$$g_2(\alpha_c) = 0.76 - 2.28\alpha_c + 3.87\alpha_c^2 - 2.04\alpha_c^3 + \frac{0.66}{(1-\alpha_c)^2} \tag{17}$$

$$\alpha_c = \frac{(a_c + HO)}{(b + HO)} \tag{18}$$

where:

a_c = critical effective-elastic crack length

b   = beam's depth

HO   = clipper gage thickness

(7) Critical effective-elastic crack length, a_c, can be derived by:

$$a_c = a_0 \frac{C_u g_2(\alpha_0)}{C_i g_2(\alpha_c)} \tag{19}$$

2.5 Non-Linear Mode I Fracture Model for Fiber Reinforced Concrete

2.5.1 Two-Parameters Fracture Model by Jenq and Shah for

Fiber Reinforced Concrete

According to Balaguru and Shah (1992), the 2 criterions of failure for fiber reinforced concrete are expressed by equations;

$$K_{Ic}^s = K_I^m + K_I^f \tag{20}$$

$$CTOD_c = CTOD_m + CTOD_f \tag{21}$$

where:

K_{Ic}^s = total stress intensity factor for composite of fiber-matrix

K_I^m = stress intensity factor for matrix

K_I^f = stress intensity factor for fiber

$CTOD_c$ = crack tip opening displacement for matrix

$CTOD_f$ = crack tip opening displacement for fiber

The values of K_I^f and $CTOD_f$ are negative.

The equation (20) and (21) are restated by Shah, et. al (1995) by postulate the failure criterion of fiber reinforced concrete based on superstition principles to be:

$$K_{Ic} = K_{Im}\left(\sigma_{mc,}a_c\right) - \int_{0}^{a_c} p_f\left(x,a_c\right)K_F\left(x,a_c\right)dx \qquad (22)$$

$$CTOD_c = CTOD_m\left(\sigma_{mc},a_c,a_0\right) - \int_{0}^{a_c} p_f\left(x,a_c\right)Q\left(\sigma_{mc},a_c,a_0\right)dx \qquad (23)$$

where:

K^s_{Ic} $\qquad$ = total stress intensity factor

K_{Im} $\qquad$ = stress intensity factor for matrix

$CTOD_c$ = total crack tip opening displacement

$CTOD_m$ $\qquad$ = crack tip opening displacement for matrix

$_{mc}$ $\qquad$ = stress on BOP[4] (*bend-over point*) that is arrested by matrix

Q $\qquad$ = Green function for closing crack along a_0 (initial crack mouth)

$\qquad$ with a unit load applied on x along the crack surface

p_f $\qquad$ = closing force on x along along the crack surface

K_F $\qquad$ = stress intensity with a unit load applied on x along the crack surface

2.5.2 Mode I Non-Linear Fracture Model by Chern, et. al

Mode I non-linear fracture model proposed by Chern, et. al (1989) for fiber reinforced concrete is covering 2 fracture criterions: (1) stress intensity factor criterion, and (2) crack tip opening displacement. The first criterion assumes that stress intensity factor lies on crack tip and then crack will grow when the values of stress intensity factor exceed its critical values. The second criterion assumes that FPZ line lies on crack tip and then stress intensity factor is going to be vanished on the tip zone. This proposed model allows the fiber existence inside the crack. Fiber traction on surface crack is a function of crack tip opening displacement so that there is no numerical iteration needed to get solution.

There is specific phenomenon in cases of fiber reinforced concrete related to fiber bridging. In fiber reinforced concrete, fiber traction will resist the crack growth and generates fiber bridging on crack surface. The fiber traction resistance causes different crack mode of fiber reinforced concrete compares to concrete without fiber reinforcement which has no fiber bridging on its surface crack. An infinite plate is shown on Figure 1 with plane strain assumption.The crack zones are divided into 3 parts: (A) traction-free zone, (B) crack

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[4] BOP, *bend-over point* is a point on stress-strain curve where crack start to be localized when the matrix contribution is maximum

bridging zone, and (C) fracture process zone. The material stress on crack tip increases from infinite values (σ_∞) of specimen boundary until ultimate tensile strength (σ_u) on nearer crack tip. For stress intensity factor criterion, there is only fiber traction zone taken account and the FPZ is ignored.
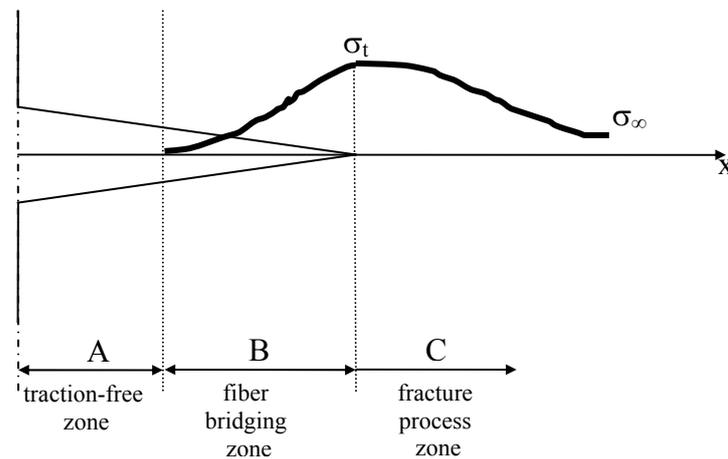

Figure 1. Crack Zones of Infinite Plate.

Mode I non-linear fracture model for fiber reinforced concrete (Chern et. al, 1989) described by Figure 2. It postulates stress intensity factor based on Dugdale mathematics model (Broek, 1982), that is expressed by:

$$K_I = 2\sqrt{\frac{a}{\pi}} \int_0^a (\sigma_\infty(s) - \sigma_c(s)) \log\left|\frac{\sqrt{a^2-x^2}+\sqrt{a^2-s^2}}{\sqrt{a^2-x^2}-\sqrt{a^2-s^2}}\right| ds \qquad (24)$$

Then, vertical crack displacement along the crack surfaces defined by:

$$v(x) = \frac{1}{\pi} \int_0^{a_e} (\sigma_\infty - \sigma_c(s) - \sigma_0) \log\left|\frac{\sqrt{a^2-x^2}+\sqrt{a^2-s^2}}{\sqrt{a^2-x^2}-\sqrt{a^2-s^2}}\right| ds \qquad (25)$$

with:

$$\sigma_c(x) = \sigma_t\left(1 - \frac{v(x)}{v(a_0)}\right)^2 \qquad \text{for} \qquad a_0 \leq x \leq a \qquad (26)$$

$$\sigma_c(x) = 0 \qquad \text{for} \qquad x\langle a_0 \quad \text{or} \quad x\rangle a \qquad (27)$$

$$\sigma_0 = 0 \qquad \text{for} \qquad x\langle a \qquad (28)$$

$$\sigma_0 = \sigma_t \qquad\qquad \text{for} \qquad a_e \geq x \rangle a \qquad\qquad (29)$$

where:

K_I = stress intensity factor

a = crack bridging zone

a_0 = traction-free zone

a_e = critical crack length

σ_∞ = stress on specimen boundary

σ_u = ultimate tensile strength

σ_c = fiber traction

σ_0 = stress on fracture process zone
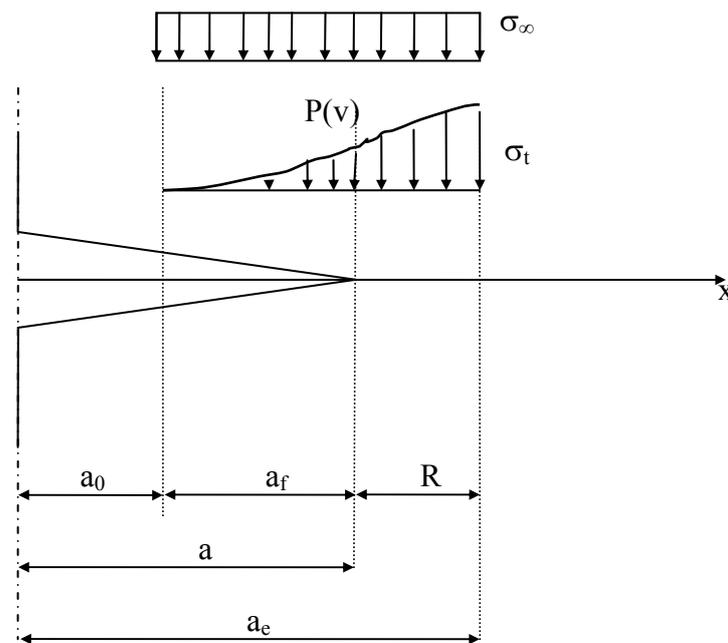
Figure 2. Crack Profile of Fiber Reinforced Concrete for Non-linear Fracture Model.

3. METHOD OF RESEARCH

This paper implements the theories of mode I non-linear fracture model on 2 cases. One case is implemented to concrete and another case is implemented to fiber reinforced concrete. Those 2 cases will derive the values of stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$ for each case. The cases are explained below.

The case 1 is a case of concrete (without fiber) notched beam specimen of mode I non-linear fracture model with specification:

Beam's depth, b = 155 mm

Beam's width, t = 85 mm

Beam's length, L = 650 mm

Beam's span, S = 4b = 620 mm

Initial notch depth, a_0 = b/3 = 51.66 mm

Measured maximum load, P_c = 900 N

Assumed *compliance* C_i = 0.0008 mm/N (based on Figure 3)

Assumed C_u = 0.000052 mm/N (based on Figure 3)

Concrete unit weigth = 2400 kg/m^3.

Curve of load-CMOD relationship described by Figure 3.
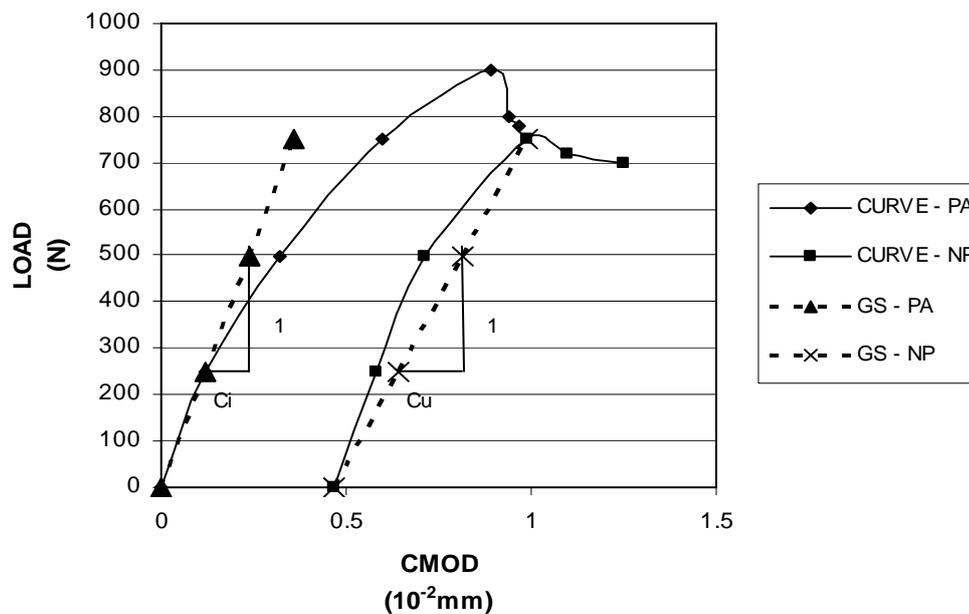

Figure 3. Load-CMOD Relationship for Case 1.

Equations (13)-(19) are used to derive the values of stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$ of Case 1.

The case 2 is infinite fiber reinforced concrete of mode I non-linear fracture model of with specification:

Crack length, a = 1 mm

Fiber bridging zone, a_f = 0.5 mm

External force, σ_∞ $= 1.19.10^{-4}$ MPa

Ultimate tensile strength, σ_t $= 3.4.10^{-4}$ MPa

Assumed FPZ length, R $= 0.2$ mm

Ultimate tensile strength for crack length comes from traction-free zone to area before FPZ with ($a_0 \leq x \leq a$). The values of stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$ of Case 2 will be calculated by equation (24)-(29), when crack length is a half of critical crack length ($x = 0.5\, a_e$).

4. RESULT AND DISCUSSION

Mode I non-linear fracture model has been used widely to derive critical stress intensity and critical crack tip opening displacement as fracture criterions for concrete and fiber reinforced concrete. The fracture criterions apply global energy equilibrium to predict fracture behavior of notched beam. It should be noted that the implementation of those fracture criterions is specific for each case. The fracture behavior of fiber reinforced concrete will be so specific because of the fiber bridging phenomenon that resists the crack growth.

According to mode I non-linear fracture models in this paper, there are some important facts that should be considered. All analytical solution of Case 1 and 2 are derived by mathematics software as a tool. Case 1 resuls the values of stress intensity factor K^S_{Ic} as 15.078 MPa mm$^{-1/2}$ and crack tip opening displacement $CTOD_c$ as 0.023 mm. Case 2 resuls the values of stress intensity factor K^S_{Ic} as 3.917.10^{-4} MPa mm$^{-1/2}$ and crack tip opening displacement $CTOD_c$ as $-1.994.10^{-4}$ mm. In general, the fiber existence gives a great influence in deriving the analytical solution.

The mode I fracture behavior of Case 1 and Case 2 are absolutely influenced by the fiber existence. When the approach of two-parameter model for concrete without fiber is applied to fiber reinforced concrete, as reviewed by Case 1, then fiber traction will be ignored and the stress intensity factor could be over-estimated. The over-estimate calculation is caused by the absence of fiber stress intensity factor. On the contrary, if the calculation that is based on Chern et. al (1989) model directly applied to fiber reinforced concrete, then the fiber-matrix composites stress intensity factor will be under-estimated because of the absence of FPZ. Concerning the calculation of crack tip opening displacement, the value may be over-estimated or under-estimated when each component, matrix or fiber, is calculated independently.

The calculation of stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$ for mode I non-linear fracture model should be taken account the existence of fiber in fiber-matrix composites. According to Balaguru dan Shah (1992)[5], Zhang dan Li (2003), maupun Shah et. al (1995)[6], it is a good solution to combine Case 1 and Case 2 calculation together to get the values of stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$ by considering the existence of fiber in fiber-matrix composites. It should be emphasized that external load and bridging force will give great contribution in determining those values mentioned above.

6. CONCLUSIONS

(1) Mode I non-linear fracture model can be used to derive the values of stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$ as fracture criterions for for concrete and fiber reinforced concrete

(2) The fracture behavior of fiber reinforced concrete is specific compare to concrete because the existence of fiber bridging phenomenon

(3) Calculating stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$, the result will be over-estimated if fiber traction is ignored, and it could be under-estimated if FPZ is ignored

(4) It is a good solution to combine Case 1 and Case 2 calculation together to get the values of stress intensity factor K^S_{Ic} and crack tip opening displacement $CTOD_c$ by considering the existence of fiber in fiber-matrix composites

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[5] See equation (20) and (21)
[6] See equation (22) and (23)

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