MODELING UNCERTAINTY'IN WAVE LOADING ON A JACKET OFFSHORE STRUCTURE

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ABSTRACT

The objective of this study is to model the uncertainties inherent in random wave excitation of a steel jacket offshore structure. A probabilistic finite element method is used as the basis for addressing these uncertainties. To assess the performance of the method, comparison of the predictions model addressing various unce these uncertainties. To assess the performance of the method, comparison of the predictions model

Keywords: probabilistic finite element, uncertainty, wave loading, jacket offshore structure

1 INTRODUCTION

The dynamic analysis of offshore structures is a complex problem not so much because of the difficulty of solving the equations of motion but rather because of the limitations of the existing knowledge for an adequate description of the sea state and the interactions of the sea, structure, and the soil. The focus of this study will be on examining the sensitivity of dynamic response using Morison formulation where the force transfer coefficients are the source of uncertainty in the predictions. The sensitivity analysis presented is focused on assessing response to uncertainties in hydrodynamic inertia force coefficient, C_M , and the drag force coefficient, C_D . The values of C_M and C_D are known to be functions of Reynolds number, relative roughness (ratio of roughness height to cylinder diameter), Keulegan-carpenter number, and beta parameter (Sarpkaya and Isaacson 1981).

The Morison equation approach is restricted to structures whose characteristic dimensions are small compared to the wave length and it is further assumed that the structure is transparent to the waves. The drag and inertia coefficients are empirical in nature and therefore experience and experiments play a crucial role in their selection. selvam and Bhattacharyya (2001) applied the nonlinear system identification method to identify the hydrodynamic coefficients. A typical example is the investigation conducted by Vengatesan et al. (2000). They presented a detailed experimental on the wave forces for a vertical rectangular cylinder for the evaluation of hydrodynamic coefficients and the

Modeling Uncertainy in Wave Loading on a Jacket Offshore Structure (Olga Pattipawaej) 61

result showed the drag and inertia coefficients are strongly affected by the variation in the aspect ratios of the cylinder.

Cassidy et al. (2001) applied the probabilistic models to the response analysis jack-up. The drag and inertia coefficients are assumed as random variables. The uncertainties are modelled using a probabilistic analysis approach and response surface method. It was found that accounting for the uncertainty in the values of a set of basic random variables significantly affected the extreme response statistics.

A simplified model of a steel jacket structured is used in time hystory dynamic analysis with a view to examining the influence of the uncertainty of drag and inertia coefficients. The model of steel jacket offshore structure described by Burke and Tighe (1971) is shown in Fig. 1. In this study the probabilistic finite element analysis is specially developed for a probabilistic analysis of a steel jacket offshore structure. An assessment of the significance of inclusion of sources of uncertainty on the distributions of displacement response behavior is made. The structural predictions displacement is compared for cases where drag and inertia coefficients are assumed constants and where they are assumed random.

2 DEVELOPMENT OF THE GOVERNING EQUATION

In this section, some details are presented on the development of a computer-based model to analyze jacket structure. The hydrodynamic forces on the structure are represented by a modified form of the Morison equation that accounts for the relative motion of the structure and the fluid kinematics. The equation relates horizontal force per unit length on a vertical cylinder to fluid and structure velocities and accelerations. The hydrodynamic force at a point on a structure due to wave loads can be expressed as

$$
F(C_D, C_M, \rho, D, u, \dot{u}, \dot{x}, t) = C_D \frac{\rho}{2} D |u - \dot{x}| (u - \dot{x}) + C_M \rho \pi \frac{D^2}{4} \dot{u}
$$
(1)

where, again C_D is the drag coefficient, C_M is the inertia coefficient, ρ is the water density, D is the structure's diameter, u and \dot{u} are the water velocity and acceleration, respectively, and \dot{x} is the structure's velocity.

Fig. 2.1 Jacket offshore model with hydrodynamic force segment after Burke and Tighe (1971)

In continuous sense, the offshore tower is a structure with an infinite number of degree of freedom. For dynamic analysis purposes, it is convenient, and generally adequate, to represent such a structure with a lumped parameter model consisting of discrete masses located at nodal points of a stiffness network. Structural damping, which is originally small, is assumed to be linearly proportional to structure velocity. External force on the structure, which is exclusively hydrodynamic, is approximated by concentrated loads at the nodal points. The matrix form of the zeroth-order equations of motion can be written in the form

Modeling Uncertainy in Wave Loading on a Jacket Offshore Stucture (Olga Pattipawaej) 63

$$
\overline{\mathbf{M}}\ddot{\overline{\mathbf{x}}}(t) + \overline{\mathbf{C}}\dot{\overline{\mathbf{x}}}(t) + \overline{\mathbf{K}}\overline{\mathbf{x}}(t) = C_D \frac{\rho}{2} D|u - \dot{x}|(u - \dot{x}) + C_M \rho \pi \frac{D^2}{4} \dot{u}
$$
(2)

where, \overline{M} is the total mass matrix of the system, C is the internal damping matrix (out of water), \overline{K} is the structural stiffness matrix, the right hand side component is the vector of external force, \bar{x} , $\dot{\bar{x}}$, and $\dot{\bar{x}}$ are the nodal displacement, velocities, and accelerations. This is the usual result used in the simulations found in the open literature.

The first-order equations require the introduction of the derivatives of the appropriate matrices with respect to the random variables of interest. Here the drag and inertia coefficients are selected as two random variables. The derivative of hydrodynamic force model presented in Eq. (2) with respect to the drag and inertia coefficients are

$$
\frac{\partial F}{\partial C_D} = \frac{\rho}{2} D |u - \dot{x}| (u - \dot{x})
$$
(3)

$$
\frac{\partial F}{\partial C_M} = \rho \pi \frac{D^2}{4} \dot{u}.
$$
(4)

<u>.</u> The analytic form of partial derivatives is preferable to numerical values although either can be used in the model. For every random variable that is considered, the number of first-order equations is simply multiplied by that number. The corresponding first-order equations are

$$
\overline{\boldsymbol{M}}\frac{\partial \ddot{\boldsymbol{x}}}{\partial C_D}\bigg|_{\overline{C_D}} + \overline{\boldsymbol{C}}\frac{\partial \dot{\boldsymbol{x}}}{\partial C_D}\bigg|_{\overline{C_D}} + \overline{\boldsymbol{K}}\frac{\partial \boldsymbol{x}}{\partial C_D}\bigg|_{\overline{C_D}} = \frac{\rho}{2}D|u - \dot{\boldsymbol{x}}|(u - \dot{\boldsymbol{x}}),\tag{5}
$$

$$
\overline{M}\frac{\partial \ddot{x}}{\partial C_M}\bigg|_{\overline{C_M}} + \overline{C}\frac{\partial \dot{x}}{\partial C_M}\bigg|_{\overline{C_M}} + \overline{K}\frac{\partial x}{\partial C_M}\bigg|_{\overline{C_M}} = \rho \pi \frac{D^2}{4}\dot{u}.
$$
\n(6)

The second-order equation when there is relationship between random variables C_D and

 C_M can be expressed as follows

$$
\overline{M} \Delta \ddot{\overline{x}}(t) + \overline{C} \Delta \dot{\overline{x}}(t) + \overline{K} \Delta \overline{x}(t) = 0, \qquad (7)
$$

where

$$
\Delta \overline{\mathbf{x}}(t) = \frac{1}{2} \frac{\partial^2 \mathbf{x}(t)}{\partial C_{D}^2} \left| \frac{\partial^2 \mathbf{x}(t)}{\partial C_D} \right|_{\overline{C_D}} \text{var}(C_D) + \frac{\partial^2 \mathbf{x}(t)}{\partial C_D \partial C_M} \left| \frac{\partial^2 \mathbf{x}(t)}{\partial C_D \partial C_M} \right|_{\overline{C_M}} \text{Cov}(C_D, C_M) + \frac{1}{2} \frac{\partial^2 \mathbf{x}(t)}{\partial C_M} \left| \frac{\partial^2 \mathbf{x}(t)}{\partial C_M} \right|_{\overline{C_M}} \text{var}(C_M).
$$
\n(8)

The systems of second order ordinary differential equations in Eqs. (2), (5), (6), and (7) can be solved using Newmark method. On the other hand, if C_D and C_M are

⁶⁴ Jurnat Teknik Sipil Volume 2 Nomor 2., April 2006 : l- ⁷³

independent each other, the second order partial derivative can be found by deriving the result of Eqs. (5) and (6). Following Eq. (7) where **b** are C_D and/or C_M , The expected value with the inclusion of second-order can be approximated.

3 JACKET STRUCTURE WAVE FORCE MODEL

The continuum forces in an exact model of a structure are approximated by ^a number of discrete forces in the theoretical model. Each force, F_j , represents the total hydrodynamic force on a segment of the structure. The equation for this force is developed by Morison et. al. in 1950 (Burke and Tighe, 1971) as follows

$$
F_j = D_{D,j} \left| \overline{u}_j - \dot{x}_j \right| \left(\overline{u}_j - \dot{x}_j \right) + D_{I,j} \dot{\overline{u}}_j \tag{9}
$$

where,

 $D_{D,j} =$ $r₂$ requirements area of segment]

 $D_{I,j} = C_M \rho \times \text{[Volume of segment]}$

 \overline{u}_j , $\dot{\overline{u}}_j$ = average velocity, acceleration over segment

 \dot{x}_j = segment velocity at mid-point.

The model allows latitude for selecting structural segment independently of nodal points to most conveniently represent forces on the structure. Structural segments may be distributed horizontally and vertically in space to represent the spatial distribution of the structure. In most cases segments are chosen to be small near the water surface where the wave flow field is strongest and spatial variations are greatest, and to be larger for greater depths where the flow field is less significant and more uniform in space.

All forces on the structure must be reduced to forces at nodes, F_i , to conform to the mathematical model, and the structure velocities and accelerations used in Eq. (9) must be calculated from nodal velocities and accelerations. These transformations are performed in a linear fashion using distribution coefficients, $w_{i,j}$. The nodal forces, F_i , are related to segment forces by:

$$
F_i = \sum_j w_{i,j} F_j. \tag{10}
$$

The segment velocities, \dot{x}_j , are related to nodal velocities by:

$$
\dot{x}_j = \sum_i w_{i,j} \dot{x}_i. \tag{11}
$$

Modeling Uncertainy in Wave Loading on a Jacket Offshore Structure (Olga Pattipawaej) 65

The distribution coefficients are determined in a simple manner as follows. When the elevation of the point of application for a force segment, denoted by the subscript k , corresponds to the elevation of a node denoted by the subscript m, then $w_{m,k} = 1$, and all other $w_{i,j} = 0$. When the elevation of the force segment is between two nodes, denoted by subscripts m and n, then the distribution coefficients, $w_{m,k}$ and $w_{n,k}$, are calculated so that the horizontal forces distributed to the two nodes have the same total force and moment as the initial force; all other $w_{i,j} = 0$.

$$
w_{n,k} = (Z_k - Z_m) / (Z_n - Z_m)
$$
\n
$$
w_{m,k} = -(Z_k - Z_n) / (Z_n - Z_m).
$$
\n(12)

4 RANDOM SEA KINEMATICS

The fluid velocities and accelerations required in the Morison equation are computed from a theoretical model to represent samples from a random sea. First, a representation of a random sea surface is generated using a specified wave amplitude power spectrum. Many parameterized wave spectra have been proposed in the literature. In this study, the JONSWAP spectrum is used (Hasselman et al. 1973). It can be expressed as

$$
S(f) = \alpha g^2 f^{-5} \exp\left[-1.25 \left(\frac{f}{f_0}\right)^{-4}\right] \gamma^{\exp\left[-\frac{(f-f_0)^2}{2\tau f_0^2}\right]}.
$$
 (14)

where, f is the frequency in hertz and f_0 is tpeak frequencies.

The JONSWAP spectrum is a five-parameter spectrum, but usually three of the parameters are held constant: $\alpha = 0.081$, γ is usually chosen as the mean value of 3.3, and $\tau = 0.07$ for $f \le f_0$ and $\tau = 0.09$ for $f > f_0$.

Following the generation random process, the desired time series of wave elevation is . obtained, i.e.,

$$
\eta\left(X_i,t\right) = \sum_{mj}^{N} a_{mj} \cos\left(2\pi f t - kX_i - \phi_{mj}\right) \tag{15}
$$

where, a_{mi} is random amplitude and ϕ_{mj} is random phase which is uniform distribution in the interval $\lceil 0, 2\pi \rceil$. The equation for surface elevation, velocities, and accelerations in long crested, sinusoidal, deepwater waves, traveling in the positive X direction, with frequency $\omega=2\pi f$ and amplitude a, are derived from linear wave theory. If the surface elevation, $\eta_o(t)$, at $X = 0$ is defined by

$$
\eta_o(t) = a e^{i\omega t} \tag{16}
$$
 and

$$
\eta_i(X_i,t) = ae^{i(\omega t - kX_i)}
$$

then the velocity and acceleration at location X_i , Z_i , below the surface are

$$
u_i(t) = a\omega e^{k(z_i - z_o)} e^{i(\omega t - kX_i)} = \omega e^{k(z_i - z_o)} \eta_i(X_i, t)
$$
\n(18)

$$
\dot{u}_i(t) = i\omega u_i(t) \tag{19}
$$

where, a is the wave amplitude, $\omega = 2\pi f$ is the wave frequency (radians/second), $k =$ ω^2 / g is the wave number for deep water, g is the gravity constant (9.807 m/sec²), Z_o is water surface elevation

An average value of velocity and acceleration over a segment is required in the time series analysis model. If the horizontal and vertical dimensions of the rectangular segment are L_X , L_z , and if X_i , Z_i are coordinates of the mid-point of the segment, then the resulting averages are:

$$
\overline{u}_i(t) = \beta_i u_i(t) \tag{20}
$$

$$
\dot{\overline{u}}_i(t) = \beta_i \dot{u}_i(t) \tag{21}
$$

where,

$$
\beta_i = \frac{\sin (k L_x / 2) \sinh (k L_z / 2)}{k L_x / 2}.
$$

5 JACKET STRUCTURE MODEL PARTICULARS

A set of numerical simulations is presented to assess the performance of the probabilistic finite element method. The Jacket Offshore structure shown in Fig. 2.1 described by Burke and Tighe (1971) was selected because of the completeness of their technical paper. The height of the structure is 328 m and the water depth is 305 m. There are 7 nodes used, whose locations can be seen in Table 5.1.

Modeling Uncertainy in Wave Loading on a Jacket Offshore Structure (OIga Pattipawaej) 67

(17)

Node	Location (m)
1	328
$\overline{\mathbf{c}}$	302
3	282
4	243
5	183
6	122
	61

Table 5.1 Nodal Location

Following the method described by Burke and Tighe (1971), the total inertia mass of the system is made up of the mass of the structure in air, including deck loads and water containing in structural members, and the 'added mass' resulting form the water surrounding the structure. Mass discretization is accomplished by allocating to each node point the mass of that portion of the structure lying closer to the given node than any of its neighbors. For this jacket structure the corresponding mass matix is

The matrix of stiffness coefficients, K_{ij} , defines a linear force deflection relationship between any two nodes of the model. A complete stiffness matrix for the structure is first formulated by conventional space-frame analysis techniques. Member properties, connectivies, and joint coordinates are used to develop a stiffiress relationship between degrees of freedom allowed for structural joints. A reduced stiffness matrix is then computed from complete matrix to correspond to the nodes for which inertial degrees of freedom have been specified. The resulting stiffness matrix is

The structural damping matrix, C , is generally not known in an explicit form. Although the solution method of this study does not require a modal analysis, the structural damping is constructed from specified modal damping ratios because the availability of experimental data on these factors. Modal damping ratios for steel structures are generally less than 0.02; the results presented here assumed a value of 0.005 in each mode. The damping matrix is

Note that the original mass, stiffness, and damping matrices as reported by Burke and Tighe (1971) have been converted to SI units and used in this study.

A 1024 second wave series is used to specify wave forces on the structures. The wave record contained time series for surface elevation at the center of the structure and velocities, and acceleration corresponding to the segment described in Table 8. It can be seen from Table 8 that the sections located more than 92 m below water surface are not subject to wave forces. Drag force constants tabulated for these sections are used only in the water structure interaction expresssion (Burke and Tighe l97l). The wave record was defined, as a sample from a JONSWAP spectrum with a significant wave height is 10 m, wave period is 10 second, and γ is chosen as 2.0.

Following 19th edition of API RP 2A Wave Force Procedures and Gulf Mexico metocean criteria, the mean of drag coefficient, C_D , and the mean inertia coefficient, C_M , can be taken to be 0.7 and 2.0, respectively (Gudmestad and Moe, 1996). The coeffrcient of variation for C_D and C_M is anticipated to be on the order of 20%. The variations of C_D

Modeling Uncertainy in Wave Loading on a Jacket Offshore Stucture (Olga Pattipawaej) 69

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and C_M depend on Keulegan-Carpenter number. If the drag coefficient increases, the inertia coefficient decreases (Sarpkaya and Isaacson 1981). Therefore, it can be assumed that there is correlation between drag and inertia coefficients (Burrows et al. 1997). When there is correlation between C_D and C_M , the value of coefficient of correlation,

$$
\rho(C_D, C_M), \text{ does not affect the calculation because } \left. \frac{\partial^2 F}{\partial C_D \partial C_M} \right|_{\overline{C_D}, \overline{C_M}} = 0.
$$

Tabte 5.2 wave Data Corresponding to segment after Burke and Tighe (1971)

70 **Jurnal Teknik Sipil** Volume 2 Nomor 2., April 2006 : 1-73

The equations of motion are integrated in a stepwise manner using the Newmark method with $\gamma = 0.5$ and $\beta = 0.125$. The method, which is an iterative one, computes the accelerations, velocities, and displacements for the structure at time t_{n+1} , based on corresponding values at time, t_n , and the acceleration at t_{n+1} . The time step increment is $\Delta t = 0.0625$.

The jacket offshore structure was analyzed using a finite element discretization of the structure. A time domain simulation was performed to obtain the displacement time history at each discrete elevation. This information was used to generate the corresponding probability density functions. By interpolation the probabilisty density functions one can generate the information as presented in Fig. 5.1.

Fig. 5.1 Probability density function of jacket offshore structure displacement response for 0^{th} order solution case

Table 5.3 presents the maximum deflections at each model node. This table indicates that when the two coefficients are correlated, the nodal displacements give exactly the same result for the $0th$ -order solution as the second order solution in Eq. (8) does not contribute. Further, this table also shows that the maximum displacements do not vary significantly.

Modeling Uncertainy in Wave Loading on a Jacket Offshore Structure (Olga Pattipawaej) 71

Tabte 5.3 Maximum Deflection

6 SUMMARY AND CONCLUSION

The probabilistic finite element methods are used to develop understanding of the response behaviour of steel jacket offshore structure. The Morison equation is used as a wave force which is composed into two parts, one due to drag and the other due to acceleration of the fluid. A reasonable assumption on the rigidity of the structure is used so that fluid-structure interaction may be disregarded. A linear wave theory is applied with the significant wave height assumed to depend on the variance of wave elevation. The uncertaintes that inherent to the jacket offshore structure are drag and inertia coefficient which do not depend on time. For the case of correlated random variables, the expected value with the inclusion of second-order solution gave exactly the same result to zeroth-order solution. This is because there is no contribution from the second-oder solution.

REFERENCES

1. Burke, B.G., and Tighe, J.T. (1971). "A time series model for dynamic behavior of offshore structures." Proc. of the Offshore Technology Conference, 775 - 788, April 19-21, Dallas, TX.

- 2. Burrows, R., Tickell, R.G., Hames, D., and Najafian, G. (1997). "Morison wave force coefficients for application to random seas." Appl. Ocean Research, 19, 183-199.
- 3. Cassidy, M.J., Houlsby, G.T., and Taylor, R. E. (2001). "Application of probabilistic models to the response analysis of jack-up." Proc. $11th$ Int. Offshore and Polar Engng Conf,,2001-JSC-153, Vol. 1, 106-113, Stavanger, Norway.
- Gudmestad, o.T., and Moe, G. (1996). "Hydrodynamic coefficients for calculation 4. of hydrodynamic loads on offshore truss structure." Marine Structures, 9, 745-758.
- Hasselmann, K. et al. (1973). "Measurements of wind-wave growth and swell decay 5. during the Joint North Sea Wave project." Deut. Hydrogr. Z., Reihe A, No. 12, 95p.
- 6. Sarpkaya, T., and Isaacson, M. (1981). Mechanics of wave forces on offshore structures. Van Nostrand Reinhold Company, New York, NY.
- 7. Selvam, R. P., and Bhattacharyya, S. K. (2001). "Parameter identification of a compliant nonlinear sDoF system in random ocean waves by reserve MISo method." Oc. Engng., 28, 1199-1223.
- 8. Vengatesan, V., Varyani, K.S., and Barltrop, N. (2000). "An experimental investigation of hydrodynamic coefficients for a vertical rectangular cylinder due to regular and random waves." Oc. Engng, 27, 291-313.

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Modeling Uncertainy in Wave Loading on a Jacket Offshore Structure (Olga Pattipawaej) 73

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Jurnal Teknik Sipil Volume 2 Nomor 2., April 2006 : 1-73