## NUMERICAL MODELLING OF PIPE LOADING USING A LINEAR INTERPOLATION METHOD

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#### ABSTRACT

The movement of pipe in sand and clay are presented, formulated and analyzed using a linear interpolation method within the framework of elastic perfectly plastic Mohr Coulomb criteria and a new two-mechanism non-associative piecewise linear Drucker Prager formulation. The two mechanisms are found when the variation of shear strength due to mean effective stress is separated from that due to plastic deviatoric strain. The modelling framework is intended to show the deformation and displacement of soil surrounding pipe. Furthermore it also describes the relationship between displacement and the force. Horizontal and vertical displacement are applied into pipe and they are modelled using finite element program, CRISP version 4. Slip element (interface) is attached in the vicinity of pipe to compare the deformation produced in this case with those without interface. Different material properties of cohesive and granular soil taken from laboratory tests are set in the model. All main parameters are recorded from lateral and horizontal loading test, as a part of the Collaborative Project on Soil/Pipe Interaction Mechanisms and Modelling. Application of the approach are presented and discussed with emphasis on identifying and optimizing some of the important factors that control the displacement of soil because of the movement of pipe. Several conclusions are drawn regarding the difficulties encountered in the numerical implementation. Illustrative numerical results for common geotechnical experiments on clay and sand using finite element software demonstrate the stable derivation of the two linear non associative Drucker Prager model.

Keyword : Pipe loading, A linear interpolation method, Deformations, Displacement, Numerical modelling, Mohr Coulomb, Drucker Prager, Interface and Forces

## 1. INTRODUCTION

The need for pipes is realized to be very important in twenty-first century. They are realized to be the most reliable and economical mode of transmitting large amounts of oil and gas, especially when they have to be pumped for large distances. However, the number of accidents have dramatically increased with the increasing number of operating pipes. The integrity of these pipes is very important due to the explosive characteristics of oil and gas. Currently, soil and water contamination due to the failure of pipelines has been raised as one of the critical issues affecting preservation of the environment.

The influence of soil movement due to pipe loading are large and very critical in geotechnical context. Buried pipelines that convey fluid could generate a high-pressure to the surrounding soil. The majority of the theoretical investigations are formulated in terms of horizontal and vertical deformations of soils. The integrity of pipes and soil is also very

important due to the explosive characteristics of oil and gas. For these reasons, intensive research efforts have been carried out on the assessment of structural integrity of pipelines.

Settlements and deformations of soils depend on types of the pipe and the surrounding soil. Clay and silt produce higher rates of alterations than sand because they have low permeability and low density. In addition, the pipe made from the steel could give bigger stresses than others. The principal requirement for integrated analyses of motions and failure in soil masses is a constitutive relationship capable of modelling the stress-strain behaviour of soil up to and beyond failure. Using a linear interpolation method, such a relationship could be employed to calculate the stresses and movements in sand up to the stage where instability impends, in conjunction with the steel pipe loading. The elastoplastic stress-strain theory explained herein was developed on the basic of results of cubical triaxial tests on cohesionless soil. This theory can be used for general three-dimensional stress conditions, but the parameters involved can be acquired from results of a series of conventional triaxial compression tests. Experimental evidence shows that it is capable of predicting the behaviour of cohesionless soil for a range of loading conditions.

The application of computer program is needed to model the loading of the pipe relating to the behaviour of soil surrounding it. The results are analyzed and utilised to give further recommendation of designing safety and suitable pipe for particular type of soil. This will bring to the optimisation of the profit from existing capital investment in oil and gas industry. The other benefit is to give the idea of identifying the important parameter of soil relating to the integrity between pipe and soil in the vicinity of it.

The problem/issue above was previously analyzed using a combination of in situ observations and numerical modelling. The finite element methods for pipe loading tests in this report simplify the pipe loading and do not integrate really a soil-pipe interaction. The first model is conducted according to Mohr Coulomb model and the other is based on the Project Constitutive Model which is done by Chan (1999) for British Gas project, a two mechanism non associative piecewise linear Drucker Prager formulation. Both of them will be used for the numerical modelling of pipe loading, which is for lateral and horizontal loading.

Finite element software, CRISP version 4, will be used to implement the soil model subroutine. There are two types of soil used in this numerical implementation, i.e. cohesive soil (clay) and cohesionless soil (sand). The purpose of using finite element program above is to show the deformation and displacement of soil surrounding pipe, and also to describe the relationship between displacement and the force. Then, the force response displacement of both of the model is analysed to explain the behaviour of the soil due to pipe loading.

## 2. LITERATURE REVIEW

## 2.1 Elastioplasticity Model for Cohesionless Soil

Simple nonlinear stress-strain relations were developed by Duncan and Chang (1970) to cover both displacement and failure in analyses. These relations are based on the generalised Hooke's law and the Mohr Coulomb failure criterion. Drucker, Gibson, and Henkel (1957) recommended the theory of plasticity which is suitable for the formula of stress-strain relations for cohesionless soil. This simple theory is useful and satisfaction if predictions are used for certain loading conditions, but stress-strain theories based entirely on assumed incremental elastic behaviour have short comings for prediction of soil response at high stress levels. Moreover, the hyperbolic relationship developed by Duncan and Chang has been found to be inadequate for accurate modelling of such effects as the influence of the intermediate principal stress, volume increases due to shear stresses, and stress-path dependency.

Drucker, Gibson, and Henkel (1957) recommended the theory of plasticity which is suitable for the formula of stress-strain relations for cohesionless soil. In addition, the normality condition from perfect plasticity should be discarded, but if the yield surface is modelled by a cap, it is not necessary to abandon the normality condition and quite acceptable predictions of stress-strain behaviour can be produced for certain loading conditions using cap models. However, if the Mohr-Coulomb criterion is used for a yield criterion, the normality condition implies far too high rates of dilation when cohesionless soil is sheared under drained conditions.

A linear interpolation model in this report is capable of realistically simulating stress-strain behaviour of sands under monotonic and cyclic, drained and undrained loading conditions. It includes features such as the softening sands at states denser than critical as they dilate in drained loading and softening of sands looser than critical in undrained loading, and the pore-water pressure increase under undrained cyclic loading.

### 2.2 Critical State Theory for Clay

The behaviour of particles becomes more dependant on the surface properties of the material as the size of the particles decrease. For clay particles the surface forces play a complex and dominant role. The actual structure of the clay minerals will not be considered here because they have very large specific surfaces due to their small size and flat shape. There is usually a negative electric charge on the crystal surface and the electro-chemical forces on these surfaces are predominant in determining particle behaviour. In certain circumstances the behaviour of clay particles are controlled by the application of suitable ion forming compounds, such as lime (Ingles, 1968).

The mathematical models are often simple for strong materials, which supports their use. For soils, the improvement of a general model of complete behaviour to encompass all soil types in all conditions would seem to be a very difficult task. The wide variety of types of soil behaviour ranging from almost fluid to dry rock-like material would not seem to lend itself to description by a single model. Here the most important variant is the water content of a given soil. The Civil Engineers who initiate most soil mechanics research are content to consider only saturated soils.

As a result some powerful theories have been evolved for saturated soils. From the beginnings of simple soil shear testing has developed the elegant and comprehensive 'Critical State Model'. Roscoe, Schofield and Wroth (1958), Drucker, Gibson & Henkel (1957), and Caladine (1963), together with insights from Jefferies (1993) built the framework called 'critical state soil mechanism' which defines why and how soil behaviour changes with density. The model developed by the Cambridge school became known as Cam Clay (Roscoe, Schofield & Thurairajah, 1963), modified Cam Clay (Burland, 1965), and Granta gravel (Schofield & Wroth, 1968). The acceptance of a comprehensive model of soil behaviour will lead to the realization of soil as one of the most important engineering materials at the disposal of man.

## 2.3 The Deformation of The Soil

In most design problems, the engineer is principally interested in the extent to which a material will deform when it is loaded. If the deformation is slight, then the extent of the deformation is of interest. Should pipe loading cause soil to deform considerably then usually only the load required to cause the behaviour is of any use. The transition between these two situations will depend on the use of the soil in question. If large deformations are unacceptable then the former holds, whereas if large deformations are allowed or required, then the latter holds. The loading behaviour of the soil is best defined by the use of finite element mehod. The concepts of effective stress are well accepted. The most general case of

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a material subjected to three-dimensional loading may be treated by three-dimensional stress theory, assuming effective stress.

## 3. A TWO MECHANISM NON ASSOCIATIVE PIECEWISE LINEAR DRUCKER PRAGER MODEL

## 3.1 The Improvement of The Original Formulation

Based on the previous report (Chan, 1999) for the British Gas project, the two mechanisms are found when the variation of shear strength due to mean effective stress is separated from that due to plastic deviatoric strain. Ali (1997) wrote the basic equation of PCM (Project Constitutive Model) which were attained directly from triaxial compression experimental curves, as follow :

$$d\varepsilon_{\nu}^{p} = \frac{dp}{A} + \frac{dq}{B}$$
(1)  
$$d\varepsilon_{s}^{p} = \frac{dq}{C}$$
(2)

Chan and Ng (1998) use direct formulation of the elasto-plastic D-matrix to improve the original formulation. This represents a two mechanism plasticity model. The first one is associated and it exhibits a pure volumetric mechanism. The other mechanism is nonassociated and it exhibits a friction type mechanism where deviatoric resistance increases with the increase of mean effective stress.

There are four different D-matrices according to q-p curve (Figure 3.1). These are happens because the two mechanisms are not active simultaneously, except in the top right hand corner region of the q-p plane. Four differences are: (i) Linear elastic within body yield surface; (ii) P-mechanism only when the stress state is moving to the right i.e. increasing p; (iii) Friction mechanism only when the stress state is moving upwards i.e. decreasing q for softening and increasing q for hardening, with elastic trial stress in the increasing q direction; (iv) Both mechanisms when the stress state is moving towards the top right i.e. increase in p and decrease in q for softening, and increases in both p and q for hardening, with elastic trial stress in the increasing q direction.



Figure 3.1 Yield Surface in q - p plane

# 3.2 Formulation of Two Mechanism Version of Project Constitutive Model

The definition of the yield surfaces and plastic potentials (Q) for this model is very useful to clarify the task of the implementation of backward Euler stress integration especially relating to the calculation of the increment of the plastic multiplier Dl. The formulation of the yield function (F) for the friction mechanism comes from the straight line linear lagrangian interpolation formula (q). The variation of shear strength is modelled using the function R(q) with respect to the Lode angle q. The basic equation of the backward Euler scheme for single mechanism is given by Crisfield (1991) (Figure 3.).



Figure 3.2 Backward Euler Return in Two dimension

$$\mathbf{F} = \mathbf{q} - \mathbf{R}(\theta) \left( \mathbf{Y} + \mathbf{H}\varepsilon_{ps} + \mathbf{M}_{f}\mathbf{p}' + \mathbf{H}'\mathbf{p}'\varepsilon_{ps} \right) = 0$$
(3)

$$R(\theta) = \frac{3 - \sin \theta'}{3 - \sin 3\theta \sin \theta'}$$
(4)

$$\mathbf{Q} = \mathbf{q} - \mathbf{D} \mathbf{p}' - \mathbf{c}_1 \tag{5}$$

$$q = \sum_{i=1}^{2} \sum_{j=1}^{2} q_{ij} \left( \varepsilon_{ps} - \varepsilon_{psi} \right) \left( p - p_{j} \right)$$
(6)

## 3.3 Validation of The New Model

The single element triaxial compression laboratory experiment chosen for constitutive model calibration. The experimental data is then simulated using four different sets of simulations using suggested or reduced elastic modulus. Harold (1999) selected three sets of experimental data for the calibration of dense behaviour for the sand used in the pipe loading experiment. They were constant mean effective stress triaxial compression tests carried out at various level of mean effective stress level, 20 kPa, 100 kPa, and 400 kPa.

Mean Effective Plastic Volumetric Stress (kPa) Strain		Elastic volumetric Strain	Total Volumetric Strain
9.9164	0.	6.198E-05	6.198E-05
22.091	0.000907	0.0001381	0.0010451
48.759	0.002226	0.0003047	0.0025307
101.66	0.003914	0.006354	0.0045494
155.16	0.005185	0.009698	0.0061548
202.28	0.006001	0.0012643	0.0072653
254.71	0.006694	0.0015919	0.0082859
308.88	0.007197	0.0019305	0.0091275
355.84	0.007528	0.002224	0.0097520
400.53	0.007809	0.0025033	0.0103123

Table 3.1 The Variation of Volumetric Strain with Mean Effective Stress

From these experimental data, Ng (1999) estimated a constant Bulk and Shear moduli of 160 Mpa and 100 Mpa respectively. These correspond to a Young's modulus of 248.28 Mpa and a poisson ratio of 0.24138, slightly low but not unreasonable for sand. A reduced set of K=80 Mpa and G=50 Mpa is also used in the finite element analysis. For the

second set, the poisson ratio remains the same but the Young's modulus is halved to a value of 124.14 Mpa. The total deviatoric strain, shown in Where K takes value of 160 Mpa. And Figure 3.4, are calculated using the following formula :

$$\varepsilon_{v} = \varepsilon_{v}^{p} + \varepsilon_{v}^{e} = \varepsilon_{v}^{p} + \frac{p}{K}$$
<sup>(7)</sup>

Where K takes value of 160 Mpa.



Figure 3.4 Mean Effective Stress versus Volumetric Strain

# 4. ELASTIC-PERFECTLY PLASTIC MOHR COULOMB MODEL

A perfectly-plastic model is a constitutive model with a fixed yield surface. For stress states represented by points within the yield surface, the behaviour is purely elastic and all strains are reversible. The new Mohr-Coulomb model yield condition is an extension of Coulomb's friction law to general states of stress. The two plastic model parameter appearing in the yield functions (f) are the well-known friction angle  $\varphi$  and the cohesion c. These yield functions together represent a hexagonal come in principal stress space. The plastic potential functions (g) contain a third plasticity parameter, the dilatancy angle  $\psi$ , required to model positive plastic volumetric strain increments (dilatancy) as actually observed for dense soils.

CRISP uses the Young's modulus as the basic stiffness modulus in the elastic model and the Mohr-Coulomb model. Standard drained triaxial tests may yield a significant rate of volume decrease at the very beginning of axial loading and a low initial value of Poisson's ratio (no). CRISPS also can handle cohesionless sands (c=0) and cohesive soil (e.g. firm and soft clay). The comparison of material parameter for Mohr Coulomb and the Two Mechanism Non Associative Piecewise Linear Drucker Prager Model is shown in Table 4.1.

Mohr Coulomb Model		A Two Mechanism Non Associative Piecewise Linear Drucker Prager Model	
E <sub>o</sub> (kN/m <sup>2</sup> )	Young's modulus at the datum elevation	\$ (°)	Angle of internal friction
ν	Poisson's ratio	D-matrix	Global stiffness matrix
c (kN/m <sup>2</sup> )	Cohesion	Δλ	Plastic multiplier
φ ( <sup>0</sup> )	Angle of internal friction	M	Critical stress state
γ <sub>bulk</sub> (kN/m²)	Bulk modulus (unit weight)	F	Yield function for friction mechanism
Уо	Datum elevation at which $E = E_0$ and $c = c_0$	Q	Plastic potential
Ψ	Dilatancy Angle	> н	Plastic moduli
m <sub>e</sub>	Rate or increase of Young's modulus with depth	) A'	Hardening modulus
m <sub>C</sub>	Rate or increase of cohesion with depth		

Table 4.1 Material Parametr for Mohr Coulomb and A Two Mechanism Non Associative Piecewise Linear Drucker Prager Model

## 5. EXPERIMENTAL WORK

The pipe used for the test has a nominal 3 m length of 300 mm OD steel pipe (Figure 5.1). It is covered with a polythelene coating and is buried horizontally under a 0.8 m cover of a granular soil (sand) or cohesive soil (clay)). Tests are conducted at two levels of compaction. Two lifting arms are attached to the pipe in order that the pipe can be pulled laterally across the rig. The initial height of the pipe above the rig floor is 0.2 m. The test tank is to be lined with a waterproof plywood lining. The waterproofing is used as a filling rather than a surface varnish. The pipe is located in the center within the side walls, with the grease covering gaps in the end of the pipe. A pipe movement of at least 100 mm is provided for the lateral displacement.

Two lateral pipe displacement rates are required; either 10.0 mm/hr, or 0.5 mm/hr. The tolerance on the displacement rate is 10 %, although both load arms should move at the same time. The pipe end seal arrangement should minimise wall friction, and help to resist

any axial movement of the pipe. The displacement is to be measured to within 0.5 mm. A suitable cell size is 300 mm. No pressure measurements on the pipe walls are made during the sand or clay tests.



Figure 5.1 Layout of Measurement Devices in Sand and Clay

## 5.1 Lateral Pipe Loading Test in Sand

The most critical condition to reach is the sand density. Procedures must be developed at each test site to measure and record the sand density achieved, and its variation around the test pipe and in the remaining volume so that the density values within the target range can be reached. There are several measurement techniques, such as : trays or tins placed in loose material, cores in compact sand, and nuclear measurement techniques which must be supported by calibration data. All of them have to represent a recognised approach for the particular sand condition. The sand must be measured in a normally dry state and its water content must be less than 1 %. It is very important to measure density during sand placement (i.e.: prior to loading application), since the soil will be disturbed during testing. Density uniformity across each layer can be checked using a portable impact tester which is exerted in compact material. A global density measurement can also be estimated by recording the total mass of sand that has been placed within the volume the test rig. There are two kind of sand placement must be considered, loosed sand and dense sand placement. They have several differences which need attention carefully.

The target density to achieve for loose sand is  $1500 \pm 100$  kg/m. Loose sand placement is carried out with as little compaction as possible. Therefore, the sand is released close to the ground, and every effort is made to avoid moving the sand within the test rig more than necessary (i.e.: walking on the surface of the sand).

Placing tins in loose sand is the suitable method to measure its density. Density tins should be evenly distributed within the test rig to define the post test density distribution. This is conducted during the filling of the test rig. It is also needed to note the position of each tin. After the test has been completed penetration tests can be carried out in a line across the test rig, using a Mackintosh Penetrometer with a modified cone. This tool is used to reach a higher blow count. Its points should lie outside the failed soil wedge above the test pipe, and should be recorded as no. blows/100 mm penetration.

Dense sand has larger density that has to be achieved than loose sand. Its goal is between 1700 and 1800 kg/m3. Compaction is performed using a standard mechanical compactor and must be applied in a constant loose layer with 50 - 100 mm thickness. Its minimum energy is 2000 kJ/m<sup>3</sup>. To get this value, the number of passes of the compactor and the foot area is recorded. A hand compaction indicator is used to check the equal compaction of the sand surrounding the test pipe so that a homogenous density throughout the test rig can be achieved.

Large core tubes with 150 mm diameter is utilized to record the post density in dense sand because it has greater volume and generates better result than smaller diameter tubes. Similar positions to the loose sand tests are applied to these cores in order to give evenly distribution within the test rig. Disturbance to the test bed is one thing that is difficult to avoid. To minimise it, an area ratio of less than 15 % are used when taking core samples. The same positions as the loose sand tests are also carried out for penetration tests, but it is conducted using a medium weight penetrometer. It is applied according to the MRSA classification in DIN 4094.

## 5.2 Lateral Pipe Loading Test in Clay

Similar with that in sands, density is the most critical condition to achieve, but there is additional requisite to reach, which is water content. Procedures are developed at each test site to achieve water contents and density values within the target range. Measurement of the soil density achieved and its variation around the test pipe are needed. Recording of density is also performed during soil placement since the soil will be disturbed during the test. They are conducted together with the soil volume test and its technique must represent a recognised approach.

Core tubes can be used to determine the post test density in clay. A proper size is 100 mm in diameter and 150mm in length with thin wall. An area ratio of less than 15 % is used to minimise the disturbance to the test bed when taking the core samples. Two types of clay, firm and soft soil, are represented by two clay strengths, which are reached by changing the water content of the sample, therefore the strength of the soil can be altered and density can be achieved.

Soft clay has higher target water content than firm clay, i.e.:  $37 \% \pm 1 \%$ , but its target dry density is lower than firm clay, i.e.:  $1280 \pm 30 \text{ kg/m}^3$ . Reaching these two parameters should produce a cohesive material with an undrained shear strength of approximately  $18 \pm 5 \text{ kPa}$ . In the other side,  $33 \% \pm 1 \%$  is the target water content for firm clay and  $1360 \pm 30 \text{ kg/m}^3$  is the target dry density to achieve for firm clay. A cohesive material with an undrained shear strength which should be produced by these two parameters is approximately  $38 \pm 10 \text{ kPa}$ . To reach the target densities for soft and firm clay, it is used approximately  $600 \text{ kJ/m}^3$  of the compaction energy per unit volume. A compaction volume of  $0.9 \text{ m}^3$  per layer is generated by assuming a rig area of 3 m x 3 m and a layer depth of 0.1 m. Based on values above, the compaction energy required per layer is:  $E_1 = 600 \times 0.9 = 540 \text{ kJ}$ .

## 6. THE IMPLEMENTATION OF MOHR COULOMB MODEL

#### **6.1 Model Performance**

Two models in this type of soil are implemented using finite element software, CRISP version 4, i.e. pipe loading in sand and clay with interface and without it. Each of them is divided into three loading condition, horizontal loads, vertical upward and downward loads. The loading conditions are modelled by applying fixities (boundary conditions) in discrete steps, called increments. There are two stage of increment block in this case, releasing fixities in the vicinity of inner side of pipe conducted in one increment and 100 load

increments are used for a finite element analysis to apply horizontal and vertical displacement into the pipe. The finite element mesh that is used in sand and clay consists of 148 linear strain triangle (LST) elements. Each of them has 7 integration points and no excess pore water pressure. The model of sand and clay has 1.328 m in height and 3 m in width and its type of material behaviour is drained since the effective parameters are applied into the model.

Pipe is built with radius of 16.4 cm and surface thickness of 12.7 mm. The centre of pipe is 36.4 cm long from the bottom of the sample. An isotropic elastic behaviour is applied to the pipe since it is rigid structure. The finite element mesh which is conducted in this structure contain 16 linear strain quadrilateral (LSQ) elements.  $3 \times 3$  integration points are provided by each of them. All elements are not consolidated for this model.

Relating to pipe loading with interface, it is used 16 slip (interface) elements, a very flat quadrilateral elements, with linear elastic properties to simulate a smooth interface between the soil and pipe. The interface (slip) element is 3-noded along its long sides, and can only be used with other linear elements (LST, LSQ and 3 node bar and beam elements), which is suitable for this model. It is attached on the ouside surface of pipe for one meter depth. The interface element is only available in 2D (not 3D) and has been validated only for plane strain analysis, whereas its performance in axisymmetric analysis is, as yet, unproven. In this cases with Mohr Coulomb model, it is used Goodman's interface element to link pipe surfaces.

Since the sample of sand is rectangular, two dimensional object and it has rounded pipe buried inside it, plane strain is used as a type of element. Cubic strain triangle is not used in this case because the model is not three dimension and it need linear strain for a finite element analysis. Additionally small strain deformations will be used due to a few large deformation can be seen in this model. As it is explained above relating to two dimensional analysis with non consolidating drained condition (LST and LSQ), all nodes in this model have two degrees of freedom because they just have horizontal and vertical displacement without no excess pore water pressure.

In order to get good results from an analysis it is used a mesh which has a large number of small finite elements concentrated in localised regions where strain gradients are relatively high. CRISP version 4 provides the unstructured mesh generator which can produce complex finite element meshes using an absolute minimum of effort. This is utilised due to the fact that the generate finite element

meshes are formulated with priority given to produce the required density of finite elements.

The unstructured generator creates the most regular elements that it can. For triangular finite elements, it tries to generate equilateral triangles and for quadrilateral elements, it tries to create squares. The size of finite elements generated within each super element is controlled by specifying grading parameters for each node in the mesh. Super node gradings are also used to inform the mesh generator how large the generated finite elements is in the vicinity of the selected super nodes. The mesh generator takes the grading values at each node and interpolates these within each super element in the mesh to gain a picture of generated element sizes for each super element.

## 6.2 Covergence Criteria of The Newton Raphson Sub-iteration

Initially in situ analysis conditions are defined in order to calculate the stresses existing in the soil before analysis begins. It is given a value of 26.56 kN/m<sup>2</sup> for effective horizontal stress ( $\sigma'_{xx}$ ), effective vertical stress ( $\sigma'_{yy}$ ), and effective of out plane stress ( $\sigma'_{zz}$ ) due to isotropic conditions. There is no need to define the void ratio and the locus yield size due to an elastic perfectly plastic model. During the initial stress analysis, sand was assumed to be linear elastic with Young's modulus (E<sub>o</sub>) of 50 Mpa and poisson ratio of 0.25. The parameter E<sub>o</sub> allow for a linear variation of stiffness and strength with depth. It can be calculated with the equation below:

$$\mathbf{E} = \mathbf{E}_{o} + \mathbf{m}_{\mathrm{F}} (\mathbf{y}_{o} - \mathbf{y}) \tag{8}$$

Because the value of  $m_E$  is equal to zero, it caused  $E = E_o$  so there is no variation of Young's modulus in this model.

For problem of pipe loading with interface, normal and shear stiffness is applied into the slip element in order to give a consistent value with the continuum materials either side of it. Normal stiffness  $(k_n)$  value is calculated from:

$$k_{-} = E(1-\nu)(1+\nu)(1-2\nu)$$
<sup>(9)</sup>

and shear stiffness stiffness value is obtained from:

 $\mathbf{k}_s = G \tag{10}$ 

where E and v is relevant to the continuum and G is shear modulus.

Quite a number of analysis have been performed. There are six problems are reported here using the experimental data. In all cases: first, Elastic perfectly plastic behaviour was used for iteration; second, the maximum number of iteration was 20; third, non consolidating linear strain elements were used; forth, a Newton Raphson method for an iterative solution was used; and finally, two stage of increment block was applied in all six cases.

Output from the analysis program comes in the form of stresses and strains for every integration point and displacement for every node in the mesh. The displacement plot for the mesh is conducted to spot regions where large deformations are relatively large .In order to produce the output of data as it exists in the mesh at an instant in time, such as : the displacement of pipe at a specific increment, it is used an instance graph. The strain and stress resulted from the displacement can be seen through the contour graph.

The differences between the displacement in any nodes are caused by several factors, such as: gravity, position of nodes, direction of displacement, the existence of slip element, and type of soil. Negative value symbolize downward displacements. Large differences of displacements between pipe loading without and with interface are happened due to there is no slip element between sand and pipe. This interface performs to reduce the stiffness between two different structures with different material properties so it has function as a deformation resistance.

## 7. NUMERICAL IMPLEMENTATION OF NON ASSOCIATIVE PIECEWISE LINEAR DRUCKER PRAGER MODEL

## 7.1 The Difference of Parameter for Each Integration Step

Theoretically, the parameters A, B (or B'), C and M are constant during one incremental step. In fact, the Newton-Raphson sub-iteration within the gauss point calculation did not attain the value of the parameters A, B, C and M change from one set to another set during the iterations due to the assumption of constant parameters in the backward Euler formation and there is the limit of the size of strain increment for each integration step. The strain increment can be subdivided into NSTEP number of steps in order to increase the chance of finding constant parameters within a single incremental step.

#### 7.2 Extrapolation of Input Data

The behaviour of the soil can be observed and identified by collecting the experimental data as much as possible. Then, they are interpolated under perfect condition to give a proper description of the soil structure. However, in practise, there are a few problems will come out linking to the using of the experimental data. Firstly, the number of the

experimental results would be limited. Then, it is also expensive and difficult to achieve representative experimental data if low and very high mean effective stress is used.

The solution is utilising the extrapolation rule if experimental data is not available for some region of the stress-strain space. However, the using of suitable extrapolation formula for the available mean effective stress should be considered carefully to avoid the utilising of the wrong parameters for the formation of the D-matrix. If this happens, it can be solved by introducing the artificial data or obtaining more experimental data.

Two common cases are frequently found in conjunction with the using of extrapolation rule. They can be defined as mean effective stress value is produced higher than the maximum available or mean effective stress value is generated lower than the minimum available, and the scatter among the slope.

## 7.3 Variation of Elastic Modulus

It is important to consider only parameter of constant Bulk and Shear modulus in oder to model the pipe loading because there will not be negative plastic strain in the experimental data as their value is high. In other words, the stiffness in the initial part of the pull-out test simulation will be much higher than was found in the actual experiment. This will create tension in granular soil so it is the key point to reduce Young's modulus by 100 times while keeping the Poisson's ratio constant.

In the active side of the pipe pull-out test, it is important to get a softer response in the soil which results in low mean effective stress. This can be obtained by using a linear variation of Young's modulus. The equation can be seen in equation (9).

$$\mathbf{E} = \mathbf{E}_{\max} \cdot \min\left[1, \frac{\mathbf{p}}{\mathbf{p}_{\max}}\right] \tag{9}$$

Where  $p_{max}$  is maximum of p value in the input data,  $E_{max}$  is the associated Young's modulus and p is the current mean effective stress. The maximum value of the Young's modulus to  $E_{max}$  is maintained using the minimum function.

## 7.4 The Improvement of the D-matrix

The D-matrix conditions can be very poor when the hardening modulus C is negative, the dilatancy is small and mobilised friction angle is high on triaxial extension side. It can achieve negative values which result in numerical difficulties. There is one good strategy for it i.e. no contractive behaviour eventhough it doesn't give significant improvements. The implementation is using the criteria of the promixity to the yield surface which is within 1 % of the distance of the yield surface from the origin of the stress space.

The initial location of the volumetric yield surface for the laboratory single element experiment is taken as the value of the initial mean effective stress of the gauss point. Then the initial value of  $\varepsilon_{ps}$  is assumed to be zero and the friction mechanism is started with the minimum size. For practical situation or model experiment when the current stress state is generated from the unloading state, it is better to use a high value of  $p_Y$  and  $\varepsilon_{ps}$ .

In some cases, the Newton Raphson sub-iteration for the backward Euler scheme within the Gauss point routine failed to converge. This can be solved using the negative value of B' i.e. dilative for all cases.

## 7.5 Comparison with Mohr-Coulomb Model

Mohr Coulomb and the new model (Chan, 2000) generate some graphs to explain and describe it. In fact, there are 12 charts can be utilised to show the force displacement response due to 12 cases implemented into the model. The comparison of force displacement response between two model can be seen in (Figure 7.1). Initially, the force responce given by soil in the vicinity of pipe increase linearly with the rising of horizontal displacement in both of numerical curve. Mohr Coulomb line starts to show a constant response after pipe moves 2 mm with the peak curve is reached in value of 30 kN roughly.





Two linear non associative Drucker Prager model curve is steeper than Mohr Coulomb model in the beginning of the run. The peak value, 500 kN, occurs when pipe approaches a 60 mm of displacement. Then the graph shows a drop of approximately 25 % from the peak

value which can not be found in Mohr Coulomb model curve. After that, it increases smoothly and after 95 mm of displacement, it climbs sharply until the end of the run. This explains that this model is able to produce softening behaviour which is also observed in experimental results(Figure 7.2)

Laboratory tests show similar force displacement response with those are produced from numerical modelling. It can be observed from load displacement curve that the peak value is 400 kN. This is much closer with the peak value for the two linear non associative Drucker Prager model, 500 kN, than that for Mohr Coulomb model. Hence, the former model is much more stable and safer than the last one.



Figure 7.2 Load Displacement Curve of BG7 Based on Laboratory Test

## 8. CONCLUSIONS

- 1. The two linear non associative Drucker Prager model has been formulated to get much more stable derivation than the one for the Mohr Coulomb model.
- 2. Convergence criteria for the Mohr Coulomb model is simpler to get than the other model because it needs higher maximum iteration and toleration to achieve a satisfying value of convergence criteria.
- 3. The soil model subroutine has been implemented using the finite element program CRISP version 4. There are 12 cases is introduced in this report applied in sand and clay using or wihout interface.
- 4. The two linear non associative Drucker Prager model is safer to be used to analyze and design underground pipe because it generates force displacement responsea which are closer to the experimental results than those which are obtained by the Mohr coulomb model:

- 5. In practical implementation, only finite element programs with equilibrium iterations could be used with the latest formulation of the two linear non associative Drucker Prager model. Otherwise, it is not required for the Mohr Coulomb model.
- 6. The observed reduction of the deformations in the numerical simulations can be related to the use of slip elements (interfaces).
- 7. It is recommended to make further comparisons with laboratory result and extend the model using the interpolation between different relative density and void ratio.

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